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Editorial Comment: A Message from the President

AT this time it seems desirable to consider briefly some things the Optical Society has accomplished during the past two years, and at the same time some of the things to which it may look forward.

Among the more outstanding accomplishments, certain ones may be listed. The Optical Society held four meetings at which in addition to the usual presentation of contributed papers, trips to points of interest, special lectures, etc., there have been symposia on up-to-date topics concerned with optics. It awarded its first Adolph Lomb medal for excellence shown in the field of optics by a worker not over 30 years of age to Dr. David L. MacAdam. It awarded its seventh Frederic Ives medal to Dr. Selig Hecht for his work in the field of vision. It established two new grades of membership, the student and the emeritus. In the former group there are now 8 members, in the latter 13 members. It broadened its scope of activity in the field of optics, and increased greatly its membership thereby, by extending its Journal coverage to include the field of spectrographic analysis. It increased its membership greatly through the activities of its membership committee. It completed, through its colorimetry committee, a comprehensive study of colorimetry, which it is hoped will appear in print within the year. It established an optical glossary project. It contributed aid to the national defense effort through its representatives on the governing board of the American Institute of Physics. It published two volumes of the *Journal of the Optical Society of America*, Vol. 30 with 666 pages; Vol. 31 with 772 pages.

Many other services are being performed by the Society through its regularly appointed committees whose labors will doubtlessly be cited as they come to or toward completion.

Looking forward to the coming year, we have the expectation that normal activities corresponding to those already listed will be continued. Special mention may be made of certain features, however. (1) We may, for instance, find it possible to extend still further the accepted scope of activity in optics to cover certain other optical activities which, for one reason or another, have not yet found a home in our society or an outlet in our journal. (2) The national defense activity as a result of society action, in view of the seriousness of the situation, seems too limited. How it can be intensified with advantage, is a matter needing thought on the part of all. If some opportunity shall appear, may it not come too late.

One other matter to be mentioned is that of the resignation of Mr. Henry F. Kurtz as treasurer, effective January 1, 1942. To him, for his efficient and careful services in that office for the past nine years, the Society is extremely grateful. It recognizes, however, that the demands made on his time, freely given, have been heavy and that it is appropriate that the task shall now be assumed by another. In his successor, we shall look for the same high ideal of service to the Society.

A. G. WORTHING,
President

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Projective Transformations of Color-Mixture Diagrams*

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(Received August 23, 1941)

Projective transformations of a standard coordinate system can be written in the form:

$$u=c_3+(e_1x+e_2y)/(c_7x+c_8y+1),$$
$$v=c_3+(e_4x+e_5y)/(c_7x+c_8y+1).$$

The shapes and relative sizes of all geometrical figures subjected to such transformations will be invariant if the values of c_7 and c_8 are invariant, and if the following quantities are also invariant:

$$(e_1e_5-e_2e_4)/(e_1e_2+e_4e_5),$$
$$(e_2^2+e_5^2)/(e_1^2+e_4^2).$$

The sizes of such figures are invariant when the numerators and denominators of these expressions are themselves invariant.

Colors having equally noticeable differences can be represented by equidistant points in some projective transformation of the standard color-mixture diagram only if in the standard diagram the separations of the points on any line representing a series of equally noticeably different colors are proportional to the square of the distance from some unique point on that line, and if the locus of such unique points for all linear series of colors is a straight line. The locus of colors equally noticeably different from any standard color can be represented adequately in the standard color-mixture diagram by an ellipse. All such ellipses can be transformed by a single projective transformation into equal-sized circles only if the common tangents of every pair of ellipses intersect on a single straight line. This line is the same as the locus of unique points for the series of colors just described. The equation of this straight line is $(c_7x+c_8y+1=0)$, where the constants are those appearing in the denominators of the successful transformation formulas. If the separations of points representing equally noticeably different colors in the standard color-mixture diagram cannot be represented adequately in the manners described, then such colors cannot be represented adequately by equidistant points in any projective transformation of the standard coordinate system.

INTEREST in the projective transformation of color-mixture diagrams has increased recently as a consequence of attempts to find transformations in which all pairs of colors having equally noticeable differences of color can be represented by pairs of equidistant points.[1-3] In a previous publication the author pointed out some influences of the various coefficients of the transformation equations on the shape of the spectrum locus in the resulting diagrams.[4] D. B. Judd has requested this further development of the conditions which must be fulfilled if two transformations are to result in spectrum loci having identical shapes.

Neither the present paper nor the previous publication[4] should be interpreted as evidence of agreement with suggestions[2,3,5] that the standard color-mixture diagram be abandoned in favor of some transformed diagram exhibiting approximately uniform spacing of equally noticeable color differences. The author considers the present ICI standard coordinate system[6,7] for colorimetry adequate for the specification of color and color tolerances,[8] and believes that the confusion which would arise from its abandonment after more than ten years of extensive use would seriously discredit the science of color-

* Communication No. 817 from the Kodak Research Laboratories.

[1] D. B. Judd, "A Maxwell triangle yielding uniform chromaticity scales," J. Opt. Soc. Am. **25**, 24–35 (1935).

[2] F. C. Breckenridge and W. R. Schaub, "Rectangular uniform-chromaticity-scale coordinates," J. Opt. Soc. Am. **29**, 370–380 (1939).

[3] F. Scofield, D. B. Judd, and R. S. Hunter, "A proposed method of designating color," Bull. A.S.T.M. 19–24 (May, 1941).

[4] D. L. MacAdam, "Projective transformation of ICI color specifications," J. Opt. Soc. Am. **27**, 294–299 (1937).

Errata.—Page 297, second of Eqs. (6), the lower case subscripts, x and z, should be upper case: $L_u=(v_Z-v_X)/D+L_w$. Next to last line of first column, and fifth and tenth lines of second column, the factor $(m_Y'+1)$ is incorrect and should be simply: m_Y'. Ninth line of second column the word *multiplied* should be *divided*. Page 298, last line of first column, the second luminosity coefficient should be $\frac{1}{3}$, rather than 2.

[5] W. D. Wright, "The sensitivity of the eye to small colour differences," Proc. Phys. Soc. (London) **53**, 93–112 (1941).

[6] D. B. Judd, "The 1931 ICI standard observer and coordinate system for colorimetry," J. Opt. Soc. Am. **23**, 359–374 (1933).

[7] A. C. Hardy and collaborators, *Handbook of Colorimetry* (The Technology Press, Cambridge, Massachusetts, 1936).

[8] D. L. MacAdam, "Representation of color tolerance on the chromaticity diagram," Am. J. Psych. **52**, 412–418 (1939).

imetry. Attempts to force the intricate mass of data concerning the noticeabilities of color differences to fit an artificially simplified scheme of representation seem to promise little prospect of complete success. There is great danger that partial success, especially if adopted as a basis for standardization, will perpetuate approximations that are inferior to the accuracy of the observational data, and that the facts revealed by observations will be obscured and forgotten.

The rectangular coordinates of the point representing a color in the standard color-mixture diagram will be designated by x and y. The rectangular coordinates of the corresponding point in a projective transformation of the standard color-mixture diagram will be designated by u and v. The equations connecting u and v with x and y may be written in the form

$$u=(c_1x+c_2y+c_3)/(c_7x+c_8y+1), \quad (1)$$

$$v=(c_4x+c_5y+c_6)/(c_7x+c_8y+1). \quad (2)$$

The coefficients, $c_1 \cdots c_8$, can be assigned any values. The dependence of the transformed shape of the color-mixture diagram on these coefficients has been described elsewhere.[4] In particular, from a geometrical interpretation of c_7 and c_8 given on pages 296 and 297 of the previous paper,[4] it is evident that all sets of transformation equations which yield any particular shape of spectrum locus must have identical denominators. This same conclusion can also be drawn from the fact that, in the spatial interpretation of the projective transformation, the line $(c_7x+c_8y+1=0)$ is the intersection of the plane (x, y) with the plane passing through the center of projection, P, and parallel to the plane (u, v). Since another projection of the spectrum locus from the center, P, can yield the same shape of spectrum locus only if the plane of the new projection is parallel to the plane (u, v), the plane through P and parallel to the new plane of projection must remain unchanged and therefore also the line of intersection $(c_7x+c_8y+1=0)$, which defines the denominator of the modified set of transformation equations. Since a change of the center of projection, P, can change the shape of the spectrum locus projected onto a plane parallel to the plane through P and the line $(c_7x+c_8y+1=0)$, the invariance of the denominators of transformation equations is not sufficient to assure invariance of the shape of the transformed spectrum locus. The chief subject of this paper is the investigation of the other conditions which, together with the invariance of the denominator, are necessary and sufficient to maintain the shape of the spectrum locus unchanged.

If two new variables, s and t, are defined, such that

$$s=x/(c_7x+c_8y+1), \quad (3)$$

$$t=y/(c_7x+c_8y+1), \quad (4)$$

then in terms of these variables the transformation Eqs. (1) and (2) become

$$u=e_1s+e_2t+c_3, \quad (5)$$

$$v=e_4s+e_5t+c_6. \quad (6)$$

The coefficients, e_1, e_2, e_4, e_5, are related to the coefficients $c_1 \cdots c_8$ by the following expressions:

$$e_1=c_1-c_3c_7, \quad (7)$$

$$e_2=c_2-c_3c_8, \quad (8)$$

$$e_4=c_4-c_6c_7, \quad (9)$$

$$e_5=c_5-c_6c_8. \quad (10)$$

The following equations* for the loci of constant s and constant t can be derived from Eqs. (5) and (6):

$$v=(e_5/e_2)u+(e_4-e_1e_5/e_2)s+c_6-c_3e_5/e_2, \quad (11)$$

$$v=(e_4/e_1)u+(e_5-e_2e_4/e_1)t+c_6-c_3e_4/e_1. \quad (12)$$

From Eq. (11) it may be seen that the loci of constant values of s are parallel straight lines in the (u, v)-plane, and the spacings of these loci are directly proportional to the differences of the corresponding values of s. Similarly, from Eq. (12) it may be seen that the loci of constant values of t are parallel straight lines, the spacings of which are directly proportional to the differences of the values of t. If invariance of the shape of the transformed spectrum locus is to be obtained, the ratio of the spacings of the loci of constant s to the spacings of the loci of constant

* The remainder of this derivation, down to condition (14), was suggested by D. B. Judd as an abridged substitute for the derivation originally submitted to him by the author. The author is grateful to Dr. Judd for his painstaking care in checking this derivation, and for this and many other suggestions.

t must be maintained as must also the angle between the lines of constant s and the lines of constant t.

The condition for invariance of this angle is of the familiar form: $(m_1-m_2)/(1+m_1m_2)$ invariant, in which m_1 and m_2 are the slopes of the lines of constant s and of constant t, respectively. Equations (11) and (12) show these slopes to be: $m_1=e_5/e_2$, and $m_2=e_4/e_1$. Therefore, the condition for the invariance of the angle between the lines of constant s and constant t becomes:

$$(e_1e_5-e_2e_4)/(e_1e_2+e_4e_5) \text{ invariant.} \quad (13)$$

The condition for invariance in the ratio of the spacings of the loci of constant s to the spacings of the loci of constant t may be found by evaluating the perpendicular distance between the s lines and the t lines, respectively, for a unit change in s or t. It is found that the distance between the line, $s=0$, and the line, $s=1$, is

$$(e_1e_5-e_2e_4)/(e_2^2+e_5^2)^{\frac{1}{2}},$$

and that the distance between the line, $t=0$, and the line, $t=1$, is

$$(e_1e_5-e_2e_4)/(e_1^2+e_4^2)^{\frac{1}{2}}.$$

The condition for invariance in the ratio of these two spacings may therefore be written:

$$(e_2^2+e_5^2)/(e_1^2+e_4^2) \text{ invariant.} \quad (14)$$

In summary, the shape of the spectrum locus is the same for all transformations for which c_7, c_8, and the expressions (13) and (14) are all invariant.

When the point, $x=0$, $y=0$ is transformed into the point $u=0$ and $v=0$, the constants c_3 and c_6 both vanish, and then

$$e_1=c_1, \quad e_2=c_2, \quad e_4=c_4, \quad \text{and} \quad e_5=c_5.$$

LIMITATIONS OF PROJECTIVE TRANSFORMATIONS

The limitations of projective transformations are not generally recognized. These limitations are pertinent to the discussion of the possibility of representing the noticeability of color differences in terms of distance in some projective transformation of color-mixture diagrams. The following analysis will demonstrate that such a representation will be possible only if the increments of distance, ΔS, along any straight line in the (x, y) diagram corresponding to equally noticeable differences of colors are proportional to the square of the distance from some point on that line.

If the line in the (x, y) diagram is inclined at the angle ϕ to the $y=0$ axis, if x_0 is the x coordinate of the intercept of that line with the $y=0$ axis, and if S is the distance along the line measured from that intercept, then

$$x=S\cos\phi+x_0, \quad (15)$$

$$y=S\sin\phi. \quad (16)$$

The values of u and v corresponding to values of S along that straight line are given by

$$u=(a_1S+a_2)/(S+a_5), \quad (17)$$

$$v=(a_3S+a_4)/(S+a_5), \quad (18)$$

where

$$a_1=(c_1\cos\phi+c_2\sin\phi)/(c_7\cos\phi+c_8\sin\phi), \quad (19)$$

$$a_2=(c_3+c_1x_0)/(c_7\cos\phi+c_8\sin\phi), \quad (20)$$

$$a_3=(c_4\cos\phi+c_5\sin\phi)/(c_7\cos\phi+c_8\sin\phi), \quad (21)$$

$$a_4=(c_6+c_4x_0)/(c_7\cos\phi+c_8\sin\phi), \quad (22)$$

$$a_5=(1+c_7x_0)/(c_7\cos\phi+c_8\sin\phi). \quad (23)$$

Equations (17) and (18) can be solved for S:

$$S=(a_2-a_5u)/(u-a_1)=(a_4-a_5v)/(v-a_3). \quad (24)$$

If T represents the distance along the corresponding line in the (u, v) diagram, and if θ is the inclination of that line to the $v=0$ axis, then $\Delta u=\Delta T\cos\theta$, and $\Delta v=\Delta T\sin\theta$. If the uniform spacing of the points representing equally noticeably different colors is accomplished by the use of the (u, v) diagram, then ΔT will be constant for all pairs of equally noticeably different colors. Along any straight line, Δu will also be constant for all such pairs of colors. The corresponding increments of S will be given by

$$\Delta S=(dS/du)\Delta u=(dS/du)\Delta T\cos\theta. \quad (25)$$

From Eq. (24) the derivative can be found:

$$dS/du=(a_1a_5-a_2)/(u-a_1)^2. \quad (26)$$

The value of u given by Eq. (17) can be substituted in Eq. (26).

$$dS/du=(S+a_5)^2/(a_1a_5-a_2). \qquad (27)^*$$

When $\tan\phi=-c_7/c_8$, then the constants, a_1, $\cdots$, a_5, become infinitely great, but the limit of dS/du, as $\tan\phi$ approaches $-c_7/c_8$, is finite, and is independent of S.

Limit, when $\tan\phi$ approaches $-c_7/c_8$,

$$dS/du=(1+c_7x_0)(c_7^2+c_8^2)^{\frac{1}{2}}/(c_1c_8-c_2c_7). \qquad (28)^*$$

Along any straight line in the (u, v) diagram, equal increments of u, given by Eq. (25), should correspond to equally noticeable differences of color. Therefore, the increments, $\Delta S=(dS/du)\Delta u$, of S along the straight line in the (x, y) diagram should correspond to equally noticeable color differences. If a color-mixture diagram having uniform chromaticity scales can be constructed, then increments of ΔS in the (x, y) diagram proportional to

$$(S+a_5)^2\cos\theta/(a_1a_5-a_2)$$

should correspond to equally noticeable color differences. This means that if the desired representation of color differences is possible in a projective transformation of the (x, y) diagram, then the curve representing ΔS as a function of S along any straight line in the (x, y) diagram must be a portion of a parabola with a vertical axis. The vertex (at which $\Delta S=0$) will occur at the value of S corresponding to the intersection of the line with the line $(c_7x+c_8y+1=0)$ which is projected to infinity by the successful transformation. For any series of colors differing along a straight line parallel to the line $(c_7x+c_8y+1=0)$, that is, for $\tan\phi=-c_7/c_8$, ΔS should be independent of S and should be equal to

$$\Delta S=(1+c_7x_0)(c_7^2+c_8^2)^{\frac{1}{2}}\Delta T\cos\theta/(c_1c_8-c_2c_7). \qquad (29)^*$$

Since observed values of ΔS cannot be zero, the observed values of ΔS along any straight line in the (x, y) diagram must be either independent of S, that is, constant, or their dependence on S must be represented by a portion of one branch of a vertical parabola, if the representation of the observed data by equal distances in some transformed diagram is to be possible. If, for any straight line in the (x, y) diagram, the dependence of observed variations of ΔS on S cannot be represented adequately by one branch of a parabola, then the results of the observations cannot be represented adequately by equal distances in any projective transformation of the (x, y) diagram. Very few series of observations permitting the calculation of comparable values of ΔS for straight lines in the (x, y) diagram have been published. Figures 5 and 6 of reference 5 and, less directly, Figs. 8–11, and 18 of reference 9, indicate that the observations cannot be represented adequately in the manner just prescribed. The adequate representation of the results of observations of color differences by equal distances in some projective transformation of the (x, y) diagram therefore seems improbable.

In some experimental arrangements it is more convenient to observe series of colors equally noticeably different from one or more standard colors. The results of such observations can be represented by the locus of points in the (x, y) diagram corresponding to the colors equally noticeably different from the standard. Such results can be represented within the accuracy of the observations by ellipses[9–12] in the (x, y) diagram. If several ellipses represent the results of comparable observations using several standard colors, then these ellipses can be transformed into equal-sized circles by some projective transformation only if the following conditions are satisfied:

In the (x, y) diagram, the common tangents of

* When θ approaches $\pm90°$, both $\cos\theta$ and $(a_1a_5-a_2)$ approach zero. In this case, therefore, the expression $(dS/du)\Delta T\cos\theta$ is indeterminate. In such cases, the increment, ΔS, can be calculated from the relation

$$\Delta S=(dS/dv)\Delta v=(dS/dv)\Delta T\sin\theta.$$

The expression for (dS/dv) can be derived from Eq. (27) by substitution of a_3 for a_1, and a_4 for a_2. In the appropriate cases, the constant value of (dS/dv) can be determined from Eq. (28) by substitution of c_4 for c_1, c_5 for c_2. The latter substitutions, and $\sin\theta$ for $\cos\theta$, permit the computation of ΔS from Eq. (29) when $\tan\phi=-c_7/c_8$, and $\theta=\pm90°$.

[9] L. C. Martin, F. L. Warburton, and W. J. Morgan, "Determination of the sensitiveness of the eye to differences in the saturation of colours," Med. Res. Council Report 188, London, 1933.

[10] D. B. Judd, "Estimation of chromaticity differences and nearest color temperature on the standard 1931 ICI colorimetric coordinate system," J. Opt. Soc. Am. **26**, 421–426 (1936).

[11] D. L. MacAdam, "Noticeability of color differences in daylight," J. Opt. Soc. Am. **30**, 657A (1940).

[12] J. W. Perry, "Colour, its measurement, discrimination and specification," Proc. Phys. Soc. (London) **53**, 272–287 (1941).

all pairs of observed ellipses must be either parallel or they must intersect on some one straight line. If the common tangents are parallel, they must also be parallel to the straight line on which the non-parallel pairs of common tangents intersect.

These conditions are consequences of the fact that the common tangents of equal-sized circles are parallel lines, and if such circles adequately represent the observations in the (u, v) diagram, then the projective transformations of these common tangents onto the x, y plane must intersect on the line $(c_7x+c_8y+1=0)$, which is projected to infinity in the u, v plane. In general, all lines of any parallel set in the (u, v) diagram correspond to lines in the (x, y) diagram which converge through a single point on the line $(c_7x+c_8y+1=0)$.

If the conditions just stated are satisfied, then the straight-line locus of the intersection points of the common tangents of the ellipses in the (x, y) diagram is the line $(c_7x+c_8y+1=0)$. The left-hand member of the equation for this line is the denominator of the transformation equations for the desired uniform chromaticity scale diagram. If the conditions cited above are not fulfilled within a tolerance that corresponds to the uncertainty of the observations, then equally noticeable color differences cannot be represented adequately by equal distances in any projective transformation of the (x, y) diagram.

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Intrinsic Brightness as a Factor in Discomfort from Glare

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(Received November 3, 1941)

IT has been demonstrated in many ways that the loss in visual efficiency due to the presence of a glare-source within the visual field depends primarily upon the illumination at the eyes rather than upon the intrinsic brightness of the glare-source.[1] Thus if a small but bright source produces the same illumination at the eyes as a large source of relatively low brightness, the reduction in visibility due to glare will be substantially the same in both cases. However, if the effect of glare is appraised from the viewpoint of ocular discomfort or ease of seeing, the foregoing generalization is not necessarily valid. In fact, it is obvious in extreme cases that the discomfort arising from glare sources within the visual field depends not only upon the illumination at the eyes but also upon the intrinsic brightness of the glare source. If the conditions of observation are so arranged that equal illuminations are produced at the eyes from sources differing in brightness and area, the smallest and consequently the brightest source will seem to be most objectionable. The purposes of the present investigation were (1) to test the validity of this introspective conclusion by means of the criterion of rate of involuntary blinking;[2] and (2) to obtain some data with respect to the magnitudes of the complementary variables of brightness and area which are definitely associated with ocular discomfort or ease of seeing.

Previously we had applied the blink-rate criterion to actual lighting conditions in three offices, with encouraging results.[3] The visual conditions involved in the present investigation are indicated diagrammatically in the accompanying illustration. The subjects were seated at a table placed in the center of a typical office illuminated by a totally indirect lighting system. The latter produced an illumination of 3.5 footcandles at the eyes and 10 footcandles upon the book to be read. The "glare source" consisted of an opal glass surface which could be uniformly illumi-

[1] Matthew Luckiesh and Frank K. Moss, *The Science of Seeing* (D. Van Nostrand Company, New York, 1937).

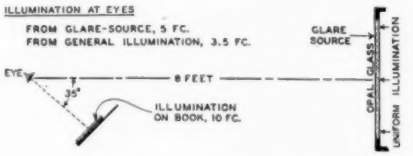
[2] Matthew Luckiesh and Frank K. Moss, "A summary of researches involving blink-rate as a criterion of ease of seeing," Illum. Eng. **35**, 19 (1940).

[3] Matthew Luckiesh and Frank K. Moss, "Brightness-contrasts in seeing," Trans. Illum. Eng. Soc. **34**, 571 (1939).

nated to specified brightnesses with its area controlled by means of suitable circular masks. As will be noted from the figure, the glare source was placed eight feet from the subject with its center at eye level, 35 degrees from the subject's line of vision while reading. The brightness and the area of the source were controlled in all cases so as to produce five footcandles, measured in vertical plane, at the eyes.

The rates of blinking during five-minute periods of reading were determined for ten adult subjects, each of whom read under each of three lighting environments on four occasions. In alternate test periods, the order of the three lighting conditions was reversed. The results are summarized in Fig. 1. These data indicate (1) that the presence of glare sources of these characteristics markedly increased the rate of blinking while reading; and (2) that the rate of blinking varied with the intrinsic brightness of the glare source notwithstanding the fact that the illumination at the eyes remained constant. It will also be noted that the factor of intrinsic brightness, for the range employed, influenced the rate of blinking to about the same extent as did the presence of the glare source itself. For example, the addition of the 150-footlambert source to the visual field increased the rate of blinking 18 percent while an increase in the brightness of the glare source from 150 footlamberts to 1770 footlamberts increased the rate of blinking from 118 to 133, or 18 percent. Furthermore, these changes in rate of blinking are highly significant when compared with their corresponding probable errors.

The experimentally determined fact that a relatively small area (20 degrees) of moderate



GLARE-SOURCE			EFFECTS
DIAMETER	VISUAL ANGLE	BRIGHTNESS	RATE OF BLINKING
NO PREVENTABLE GLARE			100 PERCENT
32 INCHES	19 DEGREES	150 FOOTLAMBERTS	118 "
10 "	6 "	1770 "	133 "
3 "	2 "	18000 "	160 "

FIG. 1. A diagrammatic representation of the lighting environments involved in the present study; and a summary of the rates of blinking while reading under these environments. The data for the smallest glare source are extrapolated.

brightness (150 footlamberts) within the visual field markedly increases the rate of blinking, and hence adversely affects ease of seeing, appears significant in lighting practice. For example, it could be predicted from such evidence that large areas of brightly illuminated ceiling, as obtained by indirect lighting, would not be conducive to the greatest degree of ocular comfort (or ease of seeing). In considering the possible deleterious effects of glare sources within the peripheral visual field, it should be recognized that while the vertical field subtends a visual angle of about 120 degrees, the horizontal or lateral field subtends a visual angle of over 200 degrees. Thus bright walls, light sources, etc., even though the latter are apparently far removed from the immediate field, may produce unfavorable visual effects.[1] Obviously the achievement of satisfactory lighting necessitates compromises between favorable and unfavorable factors intimately and complexly associated. This complexity can be unraveled only by investigations of one part at a time.

An Investigation of the Properties and Applications of the Geiger-Müller Photoelectron Counter*

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The construction of a reliable photoelectron counter with a nickel cathode and a quartz envelope is described and an account is given of comparative tests on the measurement of the relative intensities of spectral lines with the counter and by means of photographic spectrophotometry. The counter has a linear response to variations of intensity of light over a wide range. For measuring the relative intensities of spectral lines the counter tube is mounted on a spectrograph behind a slit which moves on the focal plane of the instrument so that spectral lines are measured singly. The application of the counter in spectrochemical analysis is demonstrated and comparative analyses of the same materials by use of the counter and by the method of spectrochemical analysis are given. Studies on the constancy of several spectroscopic sources are described.

INTRODUCTION

THE usual type of Geiger-Müller counter consists of a glass envelope filled with some gas such as hydrogen, air, or oxygen and containing a fine anode wire surrounded by a cathode in the form of a hollow, concentric cylinder. A discharge may be initiated by ionization produced from a β- or γ-ray. By virtue of a high potential maintained between the anode and cathode, electrons produced in the initial ionization are swept to the anode wire, and in so doing create additional ionization. Thus an avalanche of electrons arrives at the anode and produces a potential pulse that is sent through amplifying and scaling circuits and recorded.

The photoelectron counter operates much like the β- or γ-ray counter. In this counter light falling on the cathode releases a photoelectron, and this photoelectron, instead of the β- or γ-ray, institutes the avalanche mechanism. Werner[1] realized the importance of the photoelectron responses in the discharge mechanism itself, wherein photons produced by the discharge process release photoelectrons from the cathode, these electrons adding to the avalanche already under way. Thus the photoelectric properties of the cathode are intimately connected with the counter mechanism whether the photons are introduced from an external source or are produced in the tube itself.

Originally Geiger-Müller tubes, as introduced by Geiger and Müller[2] in 1928, were employed in the detection of β-, γ-, and x-rays. It was but a short time, however until the photoelectric properties were being investigated and made use of. In 1931 B. Rajewski,[3] using quartz windows, showed that counters would respond to ultraviolet light. Locher[4] in 1932 described methods of coating cathodes with alkali metals and various aniline dyes, thereby extending the response of the counters into the visible range, to around 7500A. Since then, many more investigators have described various types of photoelectron counters.

In all the reports on these photoelectron counters information concerning their reliability was either lacking or the reliability admittedly poor. By reliability is meant: (1) The existence of a flat plateau in the count-voltage characteristic curve. Such a plateau assures a counting rate independent of the applied voltage within voltage limits. (2) Constant sensitivity. The interpretation of this means that the counter response remains constant for a given light intensity. Sensitivity changes can be of a short time nature or a more gradual change over a period of time. (3) Absence of spurious counts and a low background count.

* Presented at Ninth Summer Conference on Spectroscopy and Its Applications held at Massachusetts Institute of Technology, July 21–23, 1941.

[1] S. Werner, Zeits. f. Physik **90**, 384 (1934); **92**, 705 (1934).

[2] H. Geiger and W. Müller, Physik. Zeits. **29**, 839 (1928); **30**, 489 (1929); **30**, 523 (1929).

[3] B. Rajewsky, Physik. Zeits. **32**, 121 (1931).

[4] G. Locher, Phys. Rev. **42**, 525 (1932).

It might well be expected that a photoelectron counter would have many of the properties of a photo-cell, and so it does. The two are comparable in their wave-length response and photoelectric thresholds. Both exhibit a dark count or current, especially for the alkali metals. Such effects as fatigue effects and temporary and permanent changes in sensitivity are common to both under certain conditions. One difference lies in their relative sensitivities. The Geiger-Müller photoelectron counter is capable of registering every released photoelectron, while the photo-cell must build up an appreciable current representing a large number of photoelectrons per second before it can be of use. As will be shown later, the Geiger-Müller counter is as sensitive as the electron multiplier—an extension of the photo-cell which increases its sensitivity. Thus the counter is well adapted to the detection of light of very weak intensity.

With the technique employed in this laboratory and elsewhere, reliable β- and γ-ray counters can be made. The present investigation attempts to extend the process to photoelectron counters. In particular, it is concerned with the following: (1) Construction of a reliable photoelectron counter; (2) Investigation of the photoelectric properties of the counter—such as efficiency, response to wave-length change and intensity variation, and sensitivity comparisons; (3) Applications to various problems including the study of sources and to spectrochemical analysis.

APPARATUS

I. Geiger Tube and Circuit

The anode consists of 3-mil tungsten wire stretched along the center of the tube. A spring at one end allows for expansion and prevents the wire's breaking. The cathode consists of a hollow nickel cylinder $\frac{3}{4}''$ long and $\frac{1}{2}''$ in diameter centered so that the anode falls along the axis. there is a $\frac{3}{8}''$ diameter hole through the wall of the cathode. The counter envelope is of quartz. With such a design, ultraviolet radiation, transmitted by the quartz envelope, can pass through the cathode hole and release photoelectrons from the inner cathode surface.

The tube was processed essentially as described by Duffendack, Lifschutz, and Slawsky.[5] The tube was evacuated and thoroughly outgassed with an induction furnace. By means of a neon sign transformer, a discharge was maintained in the tube and a second one connected with it in a 2-mm atmosphere of H_2. In this way, the surface of the cathode was cleaned by sputtering. Since the sputtering process deposited a metallic film on the quartz envelope, the tube was placed with the cathode hole down, and under it was placed a small piece of sheet nickel which prevented sputtering on the quartz in front of the hole. The nickel could be maneuvered externally with a magnet. During the sputtering process the anode was connected to the cathode and sputtered for a few minutes. After three or four hours sputtering, the tube was evacuated and then filled with hydrogen to a pressure of 10 cm of Hg. While still on the vacuum system, the tube was tested with a radium source for a flat plateau. If the plateau was flat, the tube was sealed off; if not, more sputtering was resorted to. Counters prepared in this way possessed a constant counting region from 800 to 1200 volts, a plateau of 400 volts.

The tube was connected to a Neher-Harper scale-of-eight circuit as designed by Lifschutz and Lawson.[6] The counter was used chiefly with the scale-of-eight circuit, but also at times it was used with a scale of one or a scale-of-sixteen circuit. Also it was used with a frequency meter, a count averaging device which gives the rate of counting at any instant instead of the individual counts.

II. Counter with Monochromator

The counter was first used with a double prism quartz monochromator. The collimator and exit arms were fixed. Spectral lines were focused on the exit slit by a rotation of the prisms produced by the turning of a screw. The screw rotation could be calibrated in terms of wave-length. The counter was mounted directly in back of the exit slit. By such an arrangement, the counter had the desirable property of being in a fixed position and having the light rays enter the counter directly (perpen-

[5] O. Duffendack, H. Lifschutz, and M. Slawsky, Phys. Rev. **52**, 1231 (1937).

[6] H. Lifschutz and J. Lawson, Rev. Sci. Inst. **9**, 83 (1938).

dicular to the plane of the counter hole). The monochromator had the disadvantage of small dispersion so that the resolution was poor for line spectra. Also, it was not adapted to the taking of photographic plates, making counter and plate comparison impossible.

III. Counter on Spectrograph

Because of the disadvantages associated with the monochromator, apparatus was designed which could be mounted interchangeably with the plateholder on a Hilger E_3 quartz spectrograph. This apparatus held a counter in position in back of an adjustable slit. The slit and counter were capable of being moved in a plane by a screw arrangement, the slit moving along the plane defined by the position of the front of the photographic plate in the plateholder. Thus when the spectral lines were in focus on a plate, the slit would move in the plane defined by the focus of the spectral lines. Thus, a single spectral line passed through the slit and into the counter. The arrangement is shown by a photograph in Fig. 1. Since the counter mounting was interchangeable with the plateholder, direct comparison and correlation with a photographic plate was possible.

IV. Constant Sources

Krypton Tube Construction

The hollow cathode discharge of a krypton lamp was extensively used in this investigation because of the richness of its ultraviolet spectrum. This type of tube yielded spark lines in the region from 1900 to 2600A. The hollow cathode material was nickel or aluminum (the aluminum being preferred because it produced less sputtering), and the ring anode material was tungsten. A quartz window was necessary. The tube operated at 40 ma or less, being supplied by a Hg rectifier unit which operated at 400 volts through a 10,000-ohm resistance in series with the tube. Intensity variations in the tube were encountered from two causes: (1) The discharge tended to extend around the end and to the outside of the cathode. A layer of mica was wrapped around the cathode to prevent this, since a glass envelope could not be fitted closely enough to be effective. (2) Sputtering of the cathode and disintegration of the mica deposited a film on the quartz window. After some hours of operation, the sputtering would so alter the discharge and darken the window that it became necessary to recondition the tube. In spite of these difficulties, the Kr lamp proved to be the best source available with the desired ultraviolet spectrum.

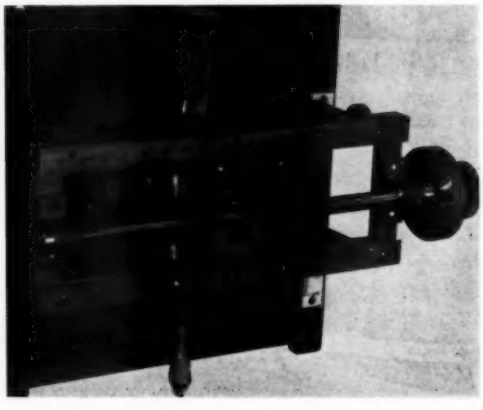

FIG. 1. Counter mounted on spectrograph.

COUNTER RESPONSE TO A KNOWN INTENSITY VARIATION

A Hg vapor lamp provided the required constant intensity source. The time of the experiment was of short enough duration so that the Hg lamp was essentially constant when used with a voltage regulator. A linear variation was accomplished by mounting the Hg lamp in a box vertically behind a vertical slit so that a portion of the positive column was exposed to the counter. A fish-tail slit could be moved horizontally across the vertical slit. By moving the fish-tail slit across the vertical slit in equal increments, the area exposed to the source was varied by linear increments. Since the positive column was uniform and of constant cross section, the intensity of the light entering the counter was likewise varied by linear increments. The counter was mounted in a metal box behind a variable slit two meters from the Hg lamp. The light from the Hg source entered the counter directly through this slit.

By noting the number of counts for linear increases in the intensity, two graphs were

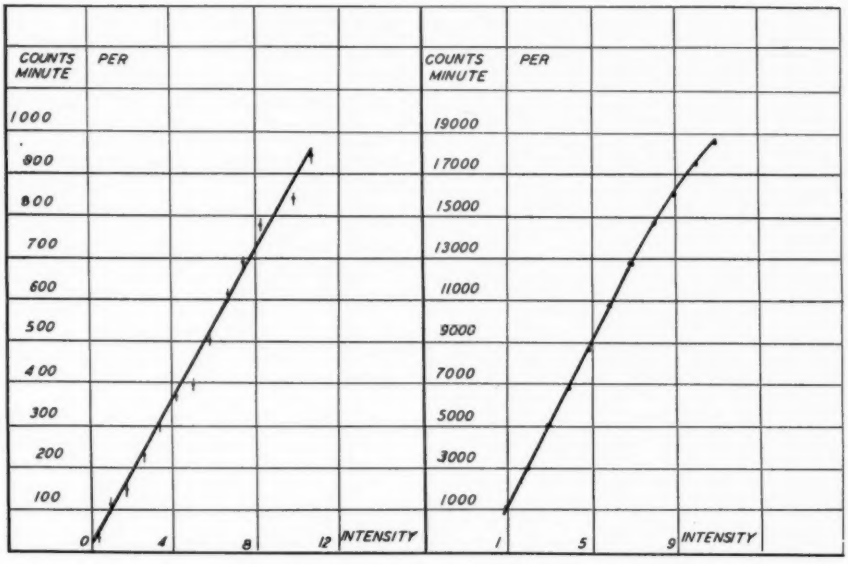

FIG. 2. Response of a Geiger-Müller counter to the linear variation in the intensity of a Hg vapor lamp.

obtained covering the range from 50 to 18,000 counts per minute. See Fig. 2. The resulting straight line curves show the highly desirable property of *a linear response in the counter for a corresponding linear change in intensity*. The sloping of the curve above 15,000 counts is due to the fact that the counting mechanism of the scale-of-eight circuit begins losing counts above this value, and not because of the non-linearity of the counter response.

CORRELATION AND COMPARISON OF THE PHOTOELECTRON COUNTER TO A PHOTOGRAPHIC PLATE. COUNTER RESPONSE AS A FUNCTION OF WAVE-LENGTH

First of all, a plate of the Kr spectrum was taken on the Hilger E_3 spectrograph. The lines on the plate were measured on a comparator. With the aid of the plate the position in the spectrograph of one of the prominent lines of the spectrum was noted and then picked up by the counter. This proved simple inasmuch as the counter slit moved along the plane of the plate, and the width of the slit, being slightly less than the width of a photographed line, permitted the spectral lines to be counted singly. Having correlated a particular line, the position of the remaining lines could be predicted for the counter by making use of the comparator measurements, since exactly the same line separation occurred on the photographic plate and on the counter scale. In this manner the position and intensity of all the lines capable of detection by the counter were recorded.

Summarizing the observations, it may be said that the counter was sensitive to radiation up to 2600A, the sensitivity falling off rapidly near the 2600 threshold. In this region the counter sensitivity was comparable to that of the photographic plate. This is a remarkable result considering that the photographic plate blackening results from an accumulation of photons over several minutes time, while the counter response is dependent on a non-accumulative, instantaneous reaction.

As a result of the counter and photographic sensitivity proceeding in opposite directions, it should be expected that counter and photographic comparisons of the relative intensities of spectral lines should agree only for lines reasonably close together. Near the counter threshold where the counter sensitivity is decreasing rapidly, agreement should occur, at best,

only for lines close together. The results of relative intensity comparisons is presented farther on.

EFFICIENCY OF THE PHOTOELECTRON COUNTER

From his calculations, Werner[1] regards as quite likely that the Geiger tube counts every electron released from the cathode surface. Other investigators concur in this opinion. This is where the Geiger counter has a great advantage over the photo-cell, which requires many released electrons to provide a detectable current. Also the insulation difficulties and dark currents limit the sensitivity of the photo-cell. If we consider the photoelectron multiplier, an extension of the photo-cell, it appears that the commercial type is far less sensitive than the photoelectron counter; but some electron multipliers of high work function have equivalent sensitivity, and if cooled in liquid air, may surpass the counter.

As to the number of quanta necessary to release a photoelectron, however, there is quite a difference of opinion. In this instance, the photoelectron counter and photo-cell should be comparable. Rajewsky[3] estimates that the counter can detect 12 quanta of 2650A light per cm^2 per sec. for a cathode coated with Zn or Cd. Barnes and Du Bridge[7] claim 10^3 quanta per cm^2 per sec. are needed at 2300A to be detected. Kreuchen[8] has made a study of the quantum efficiency of counters of various cathode materials. He calibrated the counters against a Moll thermopile, taking great pains to filter out scattered radiation by means of filters and obtained a maximum sensitivity of around 10^5 quanta/electron.

Qualitatively, the results for the nickel counter used in this research agree with the shape of Kreuchen's curve for Cu. The threshold is in the neighborhood of 2600A, and just below the threshold there is a sharp rise in sensitivity, which then approaches a constant value.

An experiment was performed to determine the order of magnitude of the efficiency of the Ni counter. The counter was exposed to the radiation of a tungsten lamp. The temperature of the lamp was determined by an optical pyrometer, and the counts per second for this temperature were noted. The distance between counter and lamp, area of filament, and area of cathode hole were known. The power radiated by the tungsten filament was available in Smithsonian Tables,[9] and the proportion of energy of a tungsten filament between 0 and 2600A was obtained from an article by Holliday.[10] This information was sufficient to determine an average value of the quanta necessary to release one photoelectron over the wave-length region for which the counter is sensitive.

Data:

$T = 2200°C$.

Distance from source to counter = 156 cm.

Power dissipated per cm^2 by W lamp at 2200°C = 38 watts.

Area of W filament = 0.78 cm^2.

Proportion of spectral energy in region from 0 to 2600A for a tungsten radiator = 4.9×10^{-6} percent of the total energy.

Counts per second at 2200°C = 162.

Counter aperture = 0.8 cm diameter.

Calculations:

1 quantum = 9.8×10^{-19} joule (at 2000A).

1 quantum/sec. = 9.8×10^{-19} watt.

$$\text{Area factor} = \frac{\text{area of counter aperture}}{\text{area of total surface of sphere of radius 156 cm}} = 1.65\times10^{-6}.$$

Total power radiated by W filament = $38\times0.8 = 30$ watts.

Energy per sec. falling on counter = $30\times1.65\times10^{-6}\times4.9\times10^{-8} = 2.4\times10^{-12}$ watt.

$$\text{Energy/count} = \frac{2.4\times10^{-12}}{162} = 1.5\times10^{-14} \text{ joule}.$$

$$\text{Quanta/count} = \frac{1.5\times10^{-14}}{9.8\times10^{-19}} = 1.5\times10^{4}.$$

The calculations show that 1.5×10^4 quanta are necessary to release one photoelectron. It must be remembered this value represents somewhat of an average, since the integrated response from 1800 to 2600A has been considered (the proportion of energy in a tungsten lamp from 0 to 1800A is negligible compared with that from 1800 to 2600A, hence it may be neglected). Recalling that the energy distribution of a W lamp drops off very rapidly towards lower wave-lengths, while the counter sensitivity increases towards the lower wave-lengths, it is

[7] S. Barnes and L. Du Bridge, Phys. Rev. **49**, 409 (1936).
[8] K. Kreuchen, Zeits. f. tech. Physik **15**, 20 (1934); **94**, 549 (1935); **97**, 625 (1935).
[9] *Fowle Smithsonian Physical Tables*, p. 321.
[10] J. Holliday, J. Opt. Soc. Am. **17**, 329 (1929).

clear that the above calculated electron yield is too high for the lower wave-lengths and too low for the higher wave-lengths. The value probably represents the true photoelectron yield for the region of 2200 or 2300A. It should also be noted that the loss of radiation due to the absorption of the two quartz windows and the air path is neglected. The calculated value of 10^4 quanta/photoelectron lies between Barnes' and Du Bridge's value of 10^3 and Kreuchen's maximum value of 10^5.

A comparison was made with recently published data from a commercial electron multiplier.[11] This multiplier is of the vacuum type, responds to visible light, and has an Ag-Cs_2O-Cs surface. This eleven-stage multiplier required 10^{-6} lumen of light input to be detectable above the dark current of the tube.

1 watt = 668 lumens.
1 quantum/sec. $= 9.8\times10^{-19}\times668 = 6.7\times10^{-16}$ lumen.
Therefore 10^{-6} lumens $= 1.5\times10^9$ quanta/sec.
Thus 1.5×10^9 quanta per sec. are necessary to be detected by the electron multiplier.

Now to make a true comparison with the nickel counter, we must consider the natural background of the counter. The natural background is around 3 counts/min. This is 0.05 count per sec. So any light source producing over 0.05 count per sec. will be detectable. Taking the previously computed value of 1.5×10^4 quanta necessary to release a photoelectron and assuming every photoelectron is counted, it becomes evident that $0.05\times1.5\times10^4 = 750$ quanta/sec. are detectable. Thus the photoelectron counter is of the order of magnitude 5×10^7 more sensitive than this eleven-stage commercial type electron multiplier.

A second comparison is made with a very recent electron multiplier described by Z. Bay.[12] It has a potassium cathode, is of the vacuum type, and responds to visible light. This tube has the advantage over the above commercial type in that it has a low dark count due to the fact that the high work function of the potassium reduced the thermal emission. The yield for this counter is estimated at 4000 photons per electron for blue light with a dark count of 2 to 10 a minute. Considering a count of 10 a minute or better as detectable, then $(10\times4000)/60 = 667$ quanta/sec. are detectable. This is about the same sensitivity as that calculated for the photoelectron counter.

A third comparison is made with a Ag-Cs_2O-Cs cathode, vacuum-type electron multiplier,[12] which is sensitive to visible light with its maximum sensitivity at 8000A. In this instance the tube is immersed in liquid air and by so doing the thermal emission ceases and the dark current becomes very small thereby increasing the sensitivity for weak radiation. For the wave-length of maximum eye sensitivity, 5300A, the estimated yield is 130 photons per electron with a dark count of 1 per minute. Considering 2 pulses per minute as detectable, it is evident $(2\times130)/60 = 4$ quanta per second are capable of detection. This is a considerably better sensitivity than the calculated 750 quanta per second for the counter.

A final word on sensitivity with regard to eye sensitivity. Observers[13,14] estimate that at the wave-length of its maximum response (5200–5300A) the eye can detect from 100 to 400 photons per second. If this be true, then the photoelectron counter has a sensitivity comparable to eye sensitivity, while the liquid-air cooled electron multiplier is more sensitive.

PHOTOELECTRON COUNTER AND PHOTOGRAPHIC COMPARISON OF THE RELATIVE INTENSITIES OF LINES IN A KRYPTON SPECTRUM

The experimental procedure follows. The light from the krypton lamp, which was operated at a constant current of 40 ma, passed through a Hilger E_3 quartz spectrograph and into the counter as described in the apparatus section. First, the intensities of various krypton lines were determined by the counter, the counts per minute being recorded. The counting time for each line was long enough to provide a total number of counts sufficient to reduce the

[11] C. Larson and H. Salinger, Rev. Sci. Inst. **11**, 226 (1940).
[12] Z. Bay, Rev. Sci. Inst. **12**, 127 (1941).
[13] R. B. Barnes and M. Czerny, Zeits. f. Physik **79**, 436 (1932).
[14] H. Buisson, J. de phys. et rad. **7**, 67 (1917); H. N. Russel, Astrophys. J. **45**, 60 (1917); and P. Reeves, Astrophys. J. **46**, 167 (1917).

TABLE I. Wave-lengths of certain lines of the krypton spectrum.

Line	Approximate Wave-Length	Line	Approximate Wave-Length
*A*4	1930A	*E*7	2167A
*A*8	1947	*E*9	2177
*A*10	1956	*E*10	2180
*A*11	1961	*E*12	2188
*B*1	1974	*E*13	2194
*B*10	2020	*E*14	2200
*C*1	2026	*E*17	2212
*C*5	2048	*F*1	2229
*C*6	2052	*F*4	2246
*C*8	2060	*F*5	2251
*C*9	2064	*F*9	2262
*C*16	2083	*F*10	2276
*C*19	2092	*F*12	2286
*C*21	2096	*G*1	2290
*D*1	2099	*G*4	2315
*D*2	2102	*G*5	2319
*D*5	2113	*G*10	2347
*D*8	2121	*G*11	2356
*D*11	2134	*H*1	2428
*E*1	2141	*H*6	2457
*E*2	2148	*H*7	2466

statistical error to a reasonable value. Knowing the intensities of the individual lines, the relative intensity of any pair was obtained by taking the ratio of their counts per minute.

Next, the counter was removed and the krypton spectrum photographed. Eastman 1–0 ultraviolet sensitized plates were used throughout. The photographic intensities were determined by standard procedure, using a stepped slit. After the step pattern and krypton lines were microphotometered, the plate density – log intensity curve was determined from the step pattern, and then the relative intensities of any desired pair of lines could be read from this curve.

This procedure illustrates the simplicity of the counter method as contrasted to the photographic method, involving plate developing, microphotometering, and step pattern calibration. Moreover, the counter maintains a linear response and so eliminates the complications arising from the non-linearity of the density – log intensity curve of the photographic plate.

Table I presents the approximate wave-lengths, as determined from a dispersion curve, of all the lines included in the subsequent experiments.

Results

Table II presents the results of the counter—photographic comparison of the relative intensities of some lines from what shall be termed krypton lamp No. I. The agreement for lines designated as *close together* is quite good, though there are a few comparisons differing by an amount up to 7 percent. However, for those comparisons in disagreement, another photographic measurement on a second spectrum (several spectra of different exposure times being required to cover the intensity ranges involved) usually gave better agreement with the counter. For those ratios designated as *fairly close* the agreement is still good in most cases, though here also are a few exceptions. Lines *not so close* give poor agreement on the whole.

It is evident that photographic and counter measurements agree only in the instance where the lines in question are reasonably close together. This was predicted previously from the qualitative observation that the sensitivities of the two methods proceeded in opposite directions.

The variable results for the counter might be attributed to the non-reliability of the counter; but since the photographic results also varied and considering the fact that photographic reliability is well established, the fault must be laid at the door of the lamp itself.

Figures 3 and 4 give an estimate of the

TABLE II. Photographic and photoelectron counter comparisons of the relative intensities of various lines of krypton lamp No. I.

Lines close together			Lines fairly close together			Lines not so close together		
Line	Phot.	Counter	Line	Phot.	Counter	Line	Phot.	Counter
*C*19/*C*21	2.92	2.95	*C*19/*C*16	4.65	4.33	*E*2/*E*7	1.88	2.12
*D*1/*C*21	2.04	2.06	*D*5/*D*2	3.15	2.80	*F*12/*F*10	3.66	3.53
*E*1/*D*11	1.25	1.27	*D*8/*D*5	1.31 1.38	1.27	*G*1/*G*4	1.41 1.55	1.60
*E*1/*E*2	1.06 1.13	1.15	*D*8/*D*11	1.69 1.59	1.46	*G*5/*G*10	2.17	2.50
*E*13/*E*12	1.36	1.37	*E*7/*E*10	1.13	1.17			
*E*14/*E*13	1.04	.975	*E*12/*E*10	1.13	1.04			
*F*4/*F*5	1.11 1.22	1.24	*E*14/*E*17	1.36	1.40			
*F*10/*F*9	1.21 1.11	1.00	*F*1/*E*17	1.99	1.78			
*F*12/*G*1	1.28 1.31	1.31	*F*1/*F*4	1.37 1.44	1.44			
*G*5/*G*4	1.44 1.33	1.37	*F*9/*F*5	1.28	1.22			
			*F*12/*F*10	3.66	3.53			
			*H*7/*H*6	3.12	2.70			

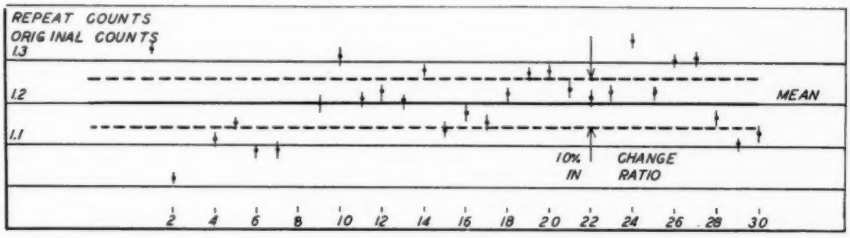

FIG. 3. The ratio of the original measurements on the intensities of various Kr lines and repeat measurements. Repeat measurements were made after 8 hours operation of the lamp from the time of the original measurements. The repeat values were lower because of sputtering on the quartz window of the lamp. The dotted lines indicate a 10 percent change in ratio.

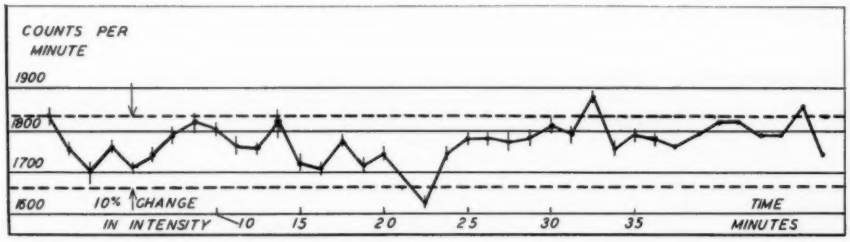

FIG. 4. The variation with time of the intensity of a particular Kr line. The dotted lines indicate a 10 percent change in intensity.

intensity fluctuation that may be expected from the lamp itself. Figure 3 is a graph of the ratios of the original count of each line taken before the photographic plate exposures to the repeat count taken after the plate exposures. The mean of the ratios of the various lines is 1.2 instead of unity. This is explained by the fact that the krypton lamp deposited a film on the window by sputtering of the cathode and disintegration of the mica around the cathode. This film consequently reduced the intensity of the lines for the repeat measurement. The graph indicates that about half the ratios lie outside a 10 percent fluctuation in the ratio (5 percent on each side of the mean). The photographic spread is also comparable to this. Figure 4 is a plot of the intensity fluctuation of a particular krypton line with time as determined from counter measurements. As shown by the plot, a fluctuation in intensity up to 10 percent should be expected.

APPLICATION OF THE PHOTOELECTRON COUNTER TO SPECTROCHEMICAL ANALYSIS

Having now gained a knowledge of the properties of the photoelectron counter, practical application was made of it to spectrochemical analysis.

A 2000-volt a.c. arc served as the source. The secondaries of two 5 kva transformers in parallel, capable of delivering 5 amperes at 2000 volts were connected to the arc gap through suitable resistance in series. The E3 Hilger spectrograph, with counter attached, was mounted one meter from the source. Light from the source fell only on the spectrograph slit, as the source was completely isolated in an adjoining room from the spectrograph.

Solution analysis was resorted to, since any element in any desired proportion could be added to the solution. The elements Zn and Pb were added in known percentages to a 10 percent Na_2CO_3 solution. Later Zn and Cd were added to a 20 percent solution of $NaNO_3$. The elements Zn, Pb, and Cd were investigated because each had a principal line in the counter sensitive region as follows: Zn, 2138A; Pb, 2170A; Cd, 2288A. The solutions were dried in an oven on $\frac{1}{4}''$ carbons which had been coated with redistilled kerosene or pyroxylin.

Preliminary tests revealed that the arc intensity was not constant enough to permit

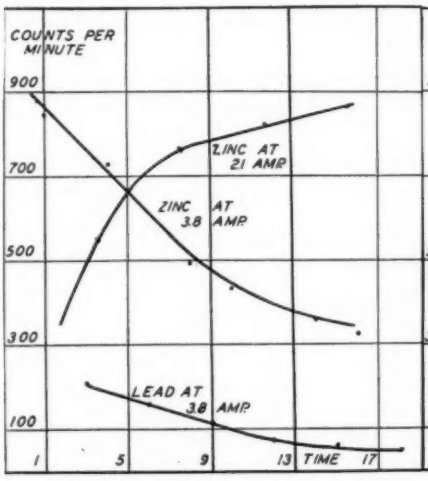

FIG. 5. The variation with time of the intensity of Zn and Pb lines from a 10 percent solution of sodium carbonate containing 0.1 percent Zn and 0.1 percent Pb. Zn 2138A, Pb 2170A 2000-v a.c. arc.

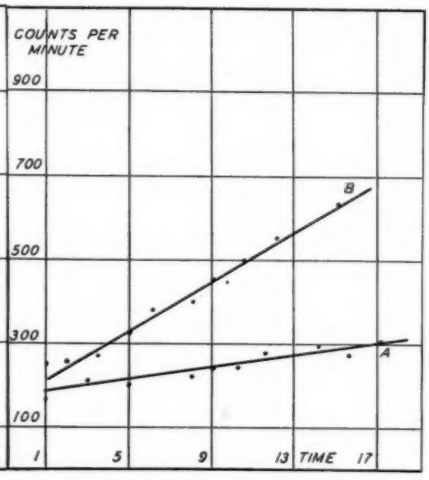

FIG. 6. Variation of the continuous background in the spectrum of a 10 percent sodium carbonate solution. 2000-v a.c. arc, 3.8 amp. Curve *A* electrodes reset after each reading. Curve *B* electrodes not reset.

either the measuring of the Zn line alone or the Zn line alternately with the Pb line which served as an internal standard (simultaneous counting on the spectrograph of the Zn and Pb lines was impossible because of the fact that the apparatus was designed to accommodate only one counter). Being the only instrument available, the monochromator was pressed into service to remedy the difficulty of the fluctuating source. It was considered probable that the difficulties connected with scattered light and low dispersion, properties inherent in the monochromator, would be minimized due to the fact that the continuous background of the spectra of the solutions was very low, and the lines appearing were few and well separated. Accordingly, another counter was mounted behind the exit slit of the monochromator. The Pb line, serving as an internal standard, passed through the spectrograph. Both lines could now be counted simultaneously. Thus, although the intensity of each line might change, under proper conditions, the relative intensity of the two should remain constant since the two lines should experience nearly identical intensity changes arising from the fluctuating source.

The next step was to determine the optimum conditions for the best reproducibility of results. Accordingly, a counter study was made of operating conditions.

Figure 5 shows typical curves of the behavior of intensity as a function of time for a source current of 3.8 amperes. Counts were recorded for a one minute period at regular intervals

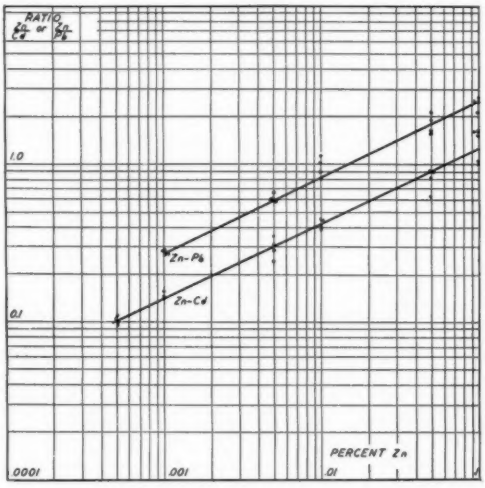

FIG. 7. Log log plot of percent Zn present in a 20 percent solution of sodium nitrate as a function of the relative intensity of Zn/Cd or Zn/Pb, Zn 2138A, Cd 2288A, Pb 2170A.

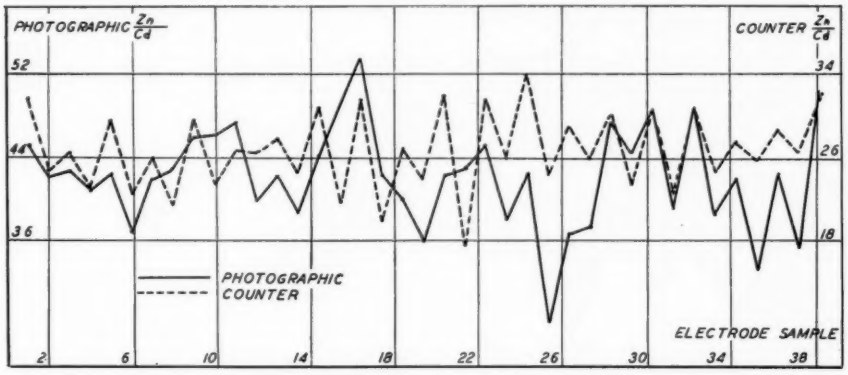

FIG. 8. Comparison of simultaneous photographic and counter determinations of Zn 2138/Cd 2276. 0.01 percent Zn and 0.01 percent Cd in 20 percent sodium nitrate solution. 2000-v a.c. arc, 2.5 amp. 5 minute exposures and counting time. Counting rate: 500 per minute for Zn; 2000 per minute for Cd. The average fluctuation of the counter determinations amounts to $\pm 7\frac{1}{2}$ percent of the amount of Zn present.

after the striking of the arc. The electrodes were not reset during the time. The intensity of both the Zn and Pb lines decreased with time indicating a rapid depletion of the elements from the source. Having observed the background intensity for the bare carbons to be considerably higher than with the solution covered carbons, a rise in background count with depletion of the electrode was suspected. Figure 6 confirms these suspicions. It represents a typical curve for change with time in background intensity near the 2138 Zn line or 2170 Pb line. The counting time was one minute, and there was no resetting of the electrodes. Figures 5 and 6 reveal, then, two opposing factors that change the intensity of lines. Resetting of the electrodes materially reduced the rise in background (Fig. 6), but it did not affect the shape of the curves in Fig. 5.

The current was reduced to 2.1 amperes in an attempt to relieve the situation of rapid source depletion. Figure 5 also shows a typical curve for this current. The electrodes were reset each time for this curve, and the counting proceeded for periods of 3 minutes. In contrast to the curve for 3.1 amperes the curve increases in intensity for several minutes, and then approaches a constant value. A more extensive study of the intensity curves for this current disclosed that the source did not begin to deplete until after a burning period of 25 minutes. At this reduced current the background proved to be much lower and constant.

As a result of the preliminary studies the operating conditions were standardized as follows: A 2.1-ampere current was adopted; the electrodes were preburned for a definite time (usually 2 minutes); the gap length was set at 0.75 mm for each reading; the counting extended over 3 minutes in order to iron out short period fluctuations and also reduce the statistical error; the elapsed time of operation was carefully noted so that the relative intensities of different pairs of electrodes could be compared under identical operation times; no corrections were made for background intensity.

Zn in percentages from 0.001 percent to 0.1 percent along with 0.1 percent Pb, as internal standard, were added to a 10 percent Na_2CO_3 solution. These solutions were dried on carbon electrodes as stated before. The counter intensities were recorded for a considerable number of electrodes at each Zn percentage under the above standard conditions.

The log Zn/Pb intensity ratio is plotted as a function of the percent Zn present in Fig. 7. The resulting straight line curve is in agreement with the well-known photographic "working curves." As is evident from the curve, the reproducibility of results at any given Zn percentage is disappointing. An intensity variation equivalent to as high as 15 percent from the mean of the amount of Zn present resulted, with the average fluctuation around 8 percent. The statistical fluctuation should amount to about 2 percent.

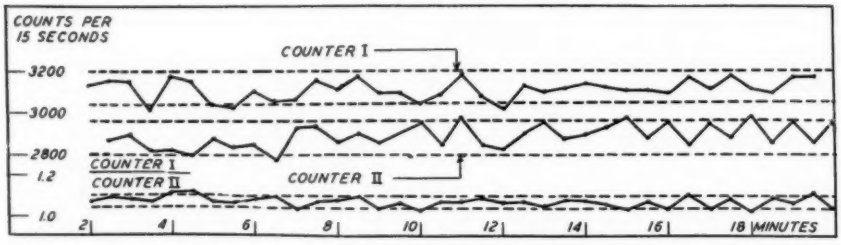

FIG. 9. Example of a constant source He lamp. Two Geiger-Müller counters counting simultaneously. The dotted lines indicate a 5 percent variation.

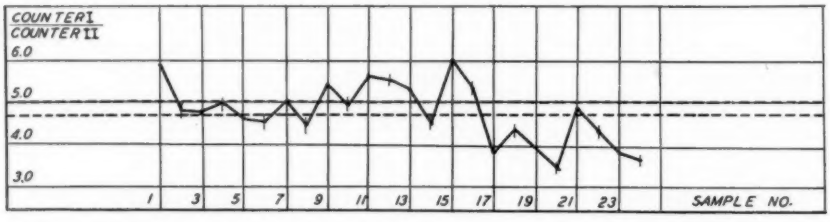

FIG. 10. The ratio of the counts made in a 2000-v a.c. arc. One counter using monochromator, the other counter using E3 Hilger quartz spectograph. 0.01 percent Zn in 20 percent sodium nitrate solution. The dotted lines indicate 5 percent variation.

It was thought Cd might prove a better internal standard than Pb in following the source fluctuation inasmuch as Cd and Zn have similar chemical properties as evidenced from the periodic table. Solutions of 20 percent $NaNO_3$, containing 0.01 percent Cd and Zn from 0.0005 percent to 1 percent, were made up. The procedure with the Zn-Cd solutions was identical with that of the Zn-Pb solutions. Figure 7 also records these results. A straight line curve again appears and again the same variability in results.

It might be well to say a word concerning the sensitivity of the counter. A 0.0005 percent Zn solution was successfully used. The lower limits to which the counter would apply with the set-up used was estimated at around 0.0001 percent. This agrees favorably with the sensitivity of the photographic method. Probably the photographic plate is more sensitive for long exposures, but the counter sensitivity could be bettered by moving the spectrograph nearer to the source than the one meter distance employed in the experiment. Here, as with the Kr studies, the counter has the advantage of a much greater intensity range than the photographic plate. The whole intensity range from 0.0005 percent to 0.1 percent Zn was covered easily by the counter.

The results of the chemical analysis were encouraging in that they revealed a straight line working curve (log intensity ratio—percent Zn curve) and sensitivity such as to be applicable to practical problems; they were disappointing in regard to reproducibility. This fault was next investigated. The trouble might lie in the source itself, in the spectrograph or monochromator, or in the counter and circuits. Each of these possibilities was investigated.

(A) Study of the Source

1. The conditions of the source operation were examined. The length of the arc gap, the position of the electrodes, the current, the counting time —all were varied. New solutions were prepared under varying conditions of acidity and with sugar added, so the consistency of the solution would be such as to adhere better to the carbons. The carbons themselves were dipped in redistilled kerosene or pyroxylin dissolved in amyl acetate to prevent the solutions from soaking into the carbons. While the factors influenced the intensity of a spectral line, no set of con-

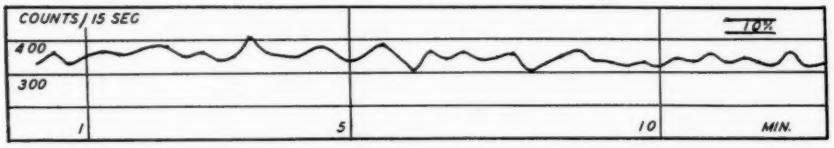

FIG. 11. The variation with time of the counts from a tungsten filament lamp lighted by a storage battery.

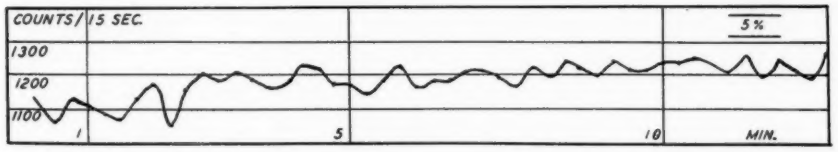

FIG. 12. Variation with time of the counts from a tungsten filament lamp lighted by a transformer.

ditions was found that would give better reproducibility.

2. A further study of the source was made by a comparison of a photographic plate and counter measurements taken simultaneously. One counter measured the Cd 2288 line by means of the monochromator; the other counter on the spectrograph measured the Zn 2138 line; and at the same time the Zn and Cd lines were photographed on a Bausch and Lomb quartz Littrow spectrograph. Throughout the period of the 5-minute plate exposure the two counters were recording. With the two counters mounted on two different instruments at different distances from the source and also with such a large wavelength separation of the Zn and Cd lines, the photographic and counter relative intensities obviously should not be expected to agree. However, the same change in relative intensities should be reflected in both. The results are presented in Fig. 8. Two exposures were taken for each pair of electrodes, the exposures and counting transpiring during intervals of 1 to 6 minutes and 6 to 11 minutes from the time the arc was struck. Both the 1–6 minute and the 7–11 minute ratios appear on the graph, accounting for the "zig-zag" pattern, since the ratio depends on the time the arc has burned. The "zig-zag" pattern is evident in both the counter and photographic results, though irregularities and variation in the degree of correlation exist.

From the experiment it may be concluded:

1. The source itself is responsible for a considerable portion of the error relative to non-reproducibility. Under this heading should be included the preparation of solutions and electrodes and choice of internal standard.

2. There are sources of error introduced by the spectrographs or counter as evidenced by the poor correlation between the counter and photographic determinations of relative intensities.

(B) Study of Counter Tubes and Circuits

The counting circuits were tested and found to be in good working order. Next followed the testing of the Geiger tubes. A constant radium source supplied rays to both counters simultaneously at the same counting rates used in the Zn-Cd analyses. The results fell within the predicted statistical error. Then the photoelectric reliability of the counters was tested by exposing them to the radiation from a He lamp, known to be of constant intensity. No spectrograph or monochromator was used, thereby eliminating that possible source of error. Figure 9 presents the results of the individual counter responses as well as the ratio of one counter to the other. Each counter confirmed the fact that the intensity of the He lamp remained constant over a long period of time, and with short time variations to have a maximum variation of 5 percent. The ratio of counter I to counter II fell almost entirely within a 5 percent limit and in good agreement with the predicted statistical fluctuation of about 3 percent, which is interpreted as meaning half the values should be within the 3 percent boundary. It should also be mentioned that reducing the intensity of the He lamp in various

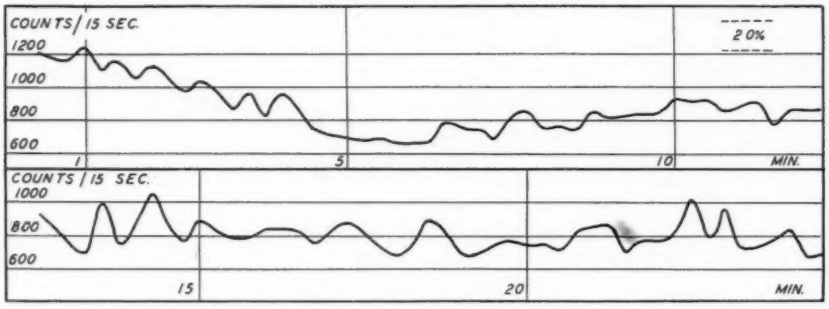

FIG. 13. Variation with time of the counts from the 2000A Cu spark line excited in a 2000-v a.c. arc.

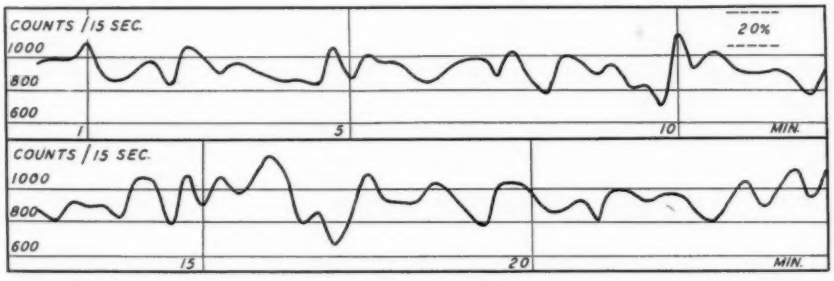

FIG. 14. Variation with time of the counts on the 2276A Cu spark line excited in a 2000-v a.c. arc.

proportions did not change the ratio of counter I to counter II, which indicates that the counters had the same change in response for a given change in the source. Obviously, the trouble was not in the counters or circuits.

(C) Study of the Spectrograph and Monochromator

For reproducible and consistent results, the monochromator and spectrograph should behave identically for a given change in intensity. To test this the 2138 Zn line was sent through both monochromator and spectrograph, and simultaneous counting was recorded. Figure 10 shows the results of 24 determinations of the ratio of counter I to counter II with both counters set on the 2138 Zn line. In contrast to the results of Fig. 9 a maximum variation of 40 percent indicated that the two instruments did not respond identically to a source common to them both. Here then was a source of trouble. Whether the trouble lay with the spectrograph or monochromator was not revealed by the experiment. It would seem likely, however, that the monochromator was at fault because of its high background (scattered radiation), poor resolution, and difficulty in accurate focusing. This result emphasizes the necessity to use high grade and appropriate instruments in this type of work. A spectrograph of high dispersion would give a greater ratio of line to background, and, if well designed, permit very little scattered light to reach the counter.

The solution of the difficulties can be found partly in an improvement of the source and partly in the construction of apparatus permitting two or more counters to be mounted on one spectrograph. It was regretted that time did not permit these suggestions to be included in the scope of this investigation.

INTENSITY STUDY OF SPECTROGRAPHIC SOURCES

Previously mentioned is one virtue of the counter—its wide range of intensity response. Another virtue arises from the fact that the counter can follow the intensity pattern of a source either concurrently or over a long time

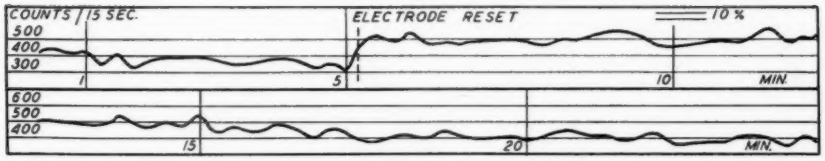

FIG. 15. Variation with time in the counts on the 2303A Cu arc line excited in a 2000-v a.c. arc.

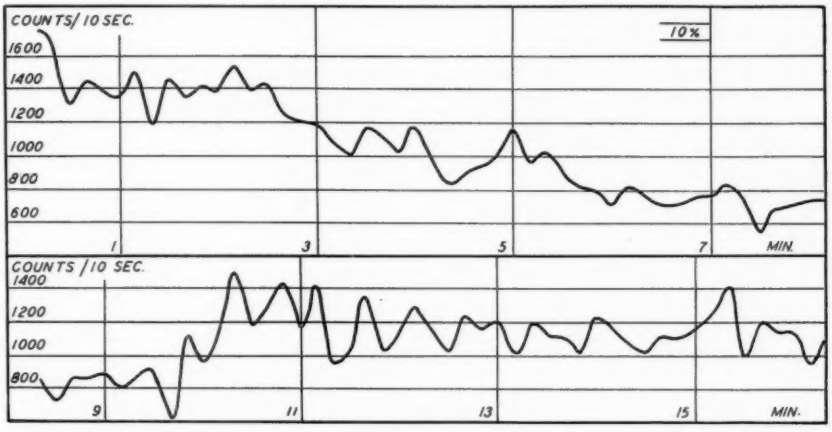

FIG. 16. Variation with time in the counts on the 2129A Cu arc line excited in a 2000-v a.c. arc.

average. The instantaneous behavior of a source can be observed by listening to the clicks of the Cenco mechanical recorder, or by using a frequency meter type of circuit. By merely recording the counts over a specified period of time, the integration or averaging of any intensity pattern may be obtained for any desired period of time.

Contrasted to this the photographic method seems cumbersome. To cover the intensity range or time of exposure range possible with the counter requires much adjustment of the distance from the source to spectrograph, the introduction of filters, and the like. Then again the photographic method is subject to errors due to non-uniformity and non-linear behavior of the photographic plate plus microphotometry errors.

All in all, the photoelectron counter appears to be an ideal method of studying the intensity patterns of light sources. Consequently, a study of various sources was undertaken. The study was not as extensive as it might have been, the emphasis being placed on illustrating the possibilities of the method.

The procedure consisted in operating various sources over a period of time and observing the intensity as given by the counter over short intervals of 10 and 15 seconds. This method gives a near instantaneous record of the source. The counting circuit was so arranged that when one mechanical counter was stopped and the counts were being recorded, another mechanical counter was counting. Both mechanical counters were connected to the same Geiger tube. Thus, a continuous record was obtained.

In addition to the sources already studied and described (Hg, He, and Kr lamps and 2000-volt a.c. arc) studies were made of a tungsten lamp, a more extensive study of 2000-volt a.c. arc, a 220-volt d.c. arc, and a condensed spark.

Tungsten Lamp

The constancy of a tungsten lamp was tested by exposing its radiation directly to a photoelectron counter. To secure a constant voltage the lamp was connected to an isolated battery. Figure 11 presents the result. In the upper right-hand corner can be noted two lines indicative of

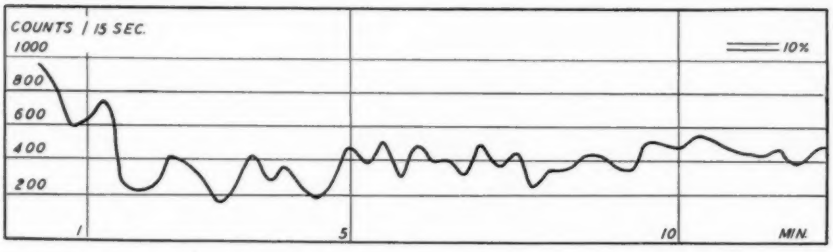

FIG. 17. Variation with time in the counts on the 2138A Zn line from a 20 percent $NaNO_3$ solution containing 0.05 percent Zn excited in a 220-v d.c. arc.

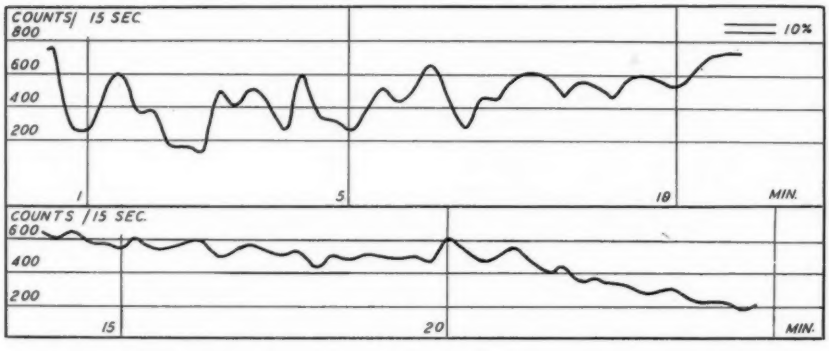

FIG. 18. Variation with time in the counts on the 2138A Zn line from a 20 percent $NaNO_3$ solution containing 0.05 percent Zn excited in a 220-v d.c. arc.

a stated percentage spread in the source fluctuation. The tungsten fluctuations are not altogether statistical, for noticeable fluctuations in the filament current of the lamp were observed.

2000-Volt A.C. Source

1. Zn in $NaNO_3$ Solution

0.05 percent Zn in a 20 percent $NaNO_3$ solution was employed in the 2000-volt a.c. arc (described in preceding section). Figure 12 presents typical results. As previously noted with current of 2.3 amperes, the intensity at first rises. This might be ascribed to a warming-up of the electrodes and the surrounding zinc vapor. After 5 minutes of operation, the intensity reaches a maximum and remains there until after 25 minutes have elapsed, at which time the source begins to deplete. It should be mentioned that the electrodes were reset every 5 minutes. From the graphs may be noted quite a fluctuation in intensity.

2. Cu Arc and Spark Lines

Figures 13 and 14 show the intensity patterns of two copper spark lines as given with metallic copper electrodes in the 2000-volt a.c. arc. An immediate decrease in sensitivity is noted (Fig. 13) for newly prepared electrodes. This decrease is caused by the formation of a copper oxide surface and persists until a considerable layer of oxide is formed. Again, considerable fluctuation in intensity occurs. The heating of the electrodes was objectionable.

Figures 15 and 16 represent two copper arc lines. In Fig. 15 old electrodes were used (hence no initial decrease in intensity), while in Fig. 16 new electrodes were employed. The apparent constancy of Fig. 15 arises from the fact that the fluctuations settle down for electrodes that have been burned a while. Also the low counting rate is misleading, since the percent fluctuation is still about the same as for Fig. 16. The arc lines seem to behave much as the spark lines.

220-Volt D.C. Arc

1. Zn in $NaNO_3$ Solution

Again the Zn solution was employed, this time in a 220-volt d.c. arc. In Figs. 17 and 18 is noted an initial large decrease in intensity then a gradual decrease in intensity until at the end of 25 minutes the source is pretty well depleted. This is in contrast to the initial increase in intensity as observed with the 2000-volt a.c. source at 2.3 amperes, but in agreement with the study at higher currents (the section on chemical analysis) of 3.5 amperes where the intensity originally decreases. There was also considerable troublesome heating of the electrodes.

2. Cu Arc and Spark Lines

The heating effect present with copper electrodes in the 220-volt d.c. arc was such as to make a study almost impossible. This undoubtedly accounts for some of the erratic results—this along with the fact that the arc hollowed out the upper electrode and formed a bead on the lower one thus making for uneven electrode separation. Figures 19 and 20 show the results on a copper arc and a spark line in the 220-volt d.c. arc. The results are erratic and not as consistent or steady as for the 2000-volt a.c. arc.

Condensed Spark

A spark source was set up consisting of a neon sign transformer which supplied 12,000 volts across a spark gap with a condenser (Leyden jar) in parallel with the gap. A voltage regulator preceded the transformer. Figures 21 and 22 represent typical results for two spark lines of copper. This uncontrolled spark source seemingly is quite a constant source, for the fluctuation falls nearly within a 5 percent spread, and a constant time average is evident. This source compares favorably to the supposedly constant tungsten lamp and is steadier than the a.c. and d.c. arcs.

These data represent only a superficial study of sources. They do indicate, however, what can be done in this line. More extensive studies should prove valuable in determining conditions for a constant source and in testing the constancy of any given source.

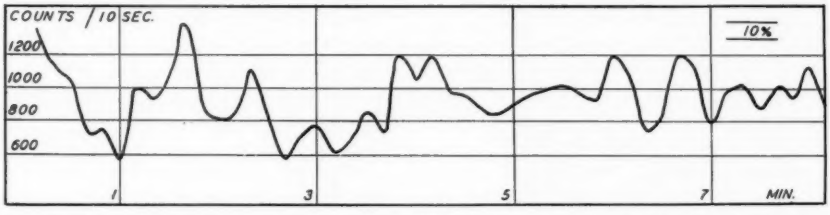

FIG. 19. Variation with time in the counts on the 2000A Cu spark line excited in the 220-v d.c. arc.

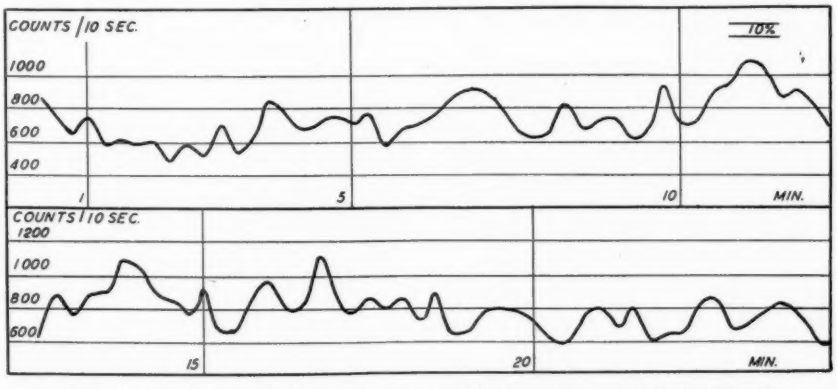

FIG. 20. Variation with time in the counts on the 2303A Cu arc line excited in the 220-v d.c. arc.

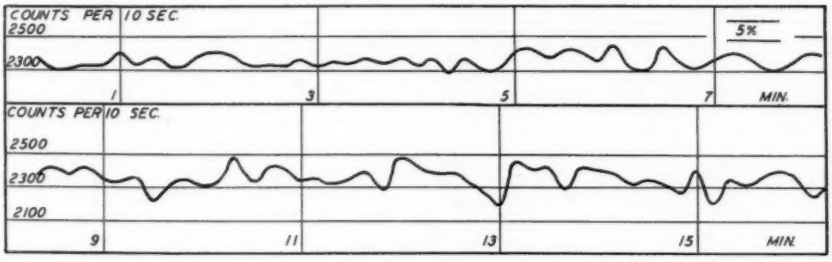

FIG. 21. Variation with time in the counts on the 2276A Cu spark line excited in a condensed spark.

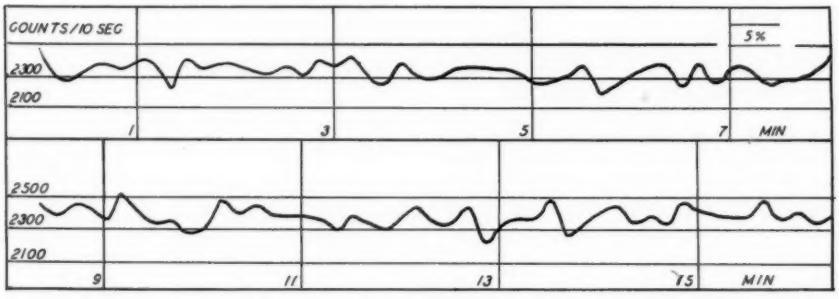

FIG. 22. Variation with time in the counts on the 2000A Cu spark line excited in a condensed spark.

CONSTRUCTION OF PHOTOELECTRON COUNTERS WITH THRESHOLDS EXTENDING INTO THE VISIBLE

One of the serious limitations on the reliable nickel counters arises from the fact that their photoelectric threshold lies in the ultraviolet around 2600A. There are a limited number of elements having prominent lines in the region between 2600A and the vacuum region. Another trouble inherent in this region of wave-lengths is the absorption of radiation by quartz and by air. This absorption by quartz becomes noticeable at 2500A, and at 1800A practically no radiation is transmitted through a meter of air. If a counter could be constructed that would respond up to 4000A, it would cover much of the region employed by the spectroscopist and consequently would be applicable to many problems that interest him. Accordingly, attempts were made to construct a counter that would respond to visible light and at the same time be reliable.

Geiger-Müller tubes were made with cathodes of aluminum (of extraordinary purity), alumel alloy (97 percent Ni, 3 percent Al), and cuprous oxide on copper. Hydrogen, oxygen, nitrogen, and argon were tried as filling gases as well as mixtures of alcohol vapor and several of these gases. Metal films were deposited on the glass walls of the tubes and held at various potentials with respect to the other electrodes of the tube. While the alumel and cuprous oxide counters filled with hydrogen were sensitive to visible light, they exhibited fatigue effects and changes of sensitivity with time and with exposure to strong light. This behavior made them unsuitable for quantitative work.

A peculiar effect was noted with the alcohol-hydrogen tube. Instead of the usual response to light, the reverse effect was noted; in darkness the tube choked the counter, and as more light was admitted the counting rate decreased and finally ceased altogether in full illumination from a light bulb or daylight. The tube was sealed off from the vacuum system and still responds in this manner. This effect was also noted once with an alumel tube and once with a Cu_2O tube.

Interference Phenomena with a Moving Medium. II

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(Received October 17, 1941)

INTRODUCTION

IN a previous paper under this title[1] experiments were described on the interfering systems of ripples set up by intermittent air jets on a mercury surface when the jets and surface were in relative motion. Invariance of the interference patterns with motion was demonstrated, upon taking into account the Fitzgerald-Larmor-Lorentz contractions. In that study the surface was viewed normally, which permitted observation of the entire interference patterns. The method did not, however, lend itself well to evaluation of the amplitudes of the waves. It was concluded from the results of tests with a variety of illumination conditions, that the *amplitudes* of the waves fore and aft of moving sources were alike. Before, however, pursuing the consequences of this conclusion, it was felt desirable to make further experiments by a method of observation which would give unequivocal results on this question of amplitudes. The experiments described below were accordingly made by a method, the salient feature of which is the use of the image of a linear light source incident at a large angle to the normal, the reflection of which from the mercury surface gives an actual trace or profile of the wave form.

APPARATUS

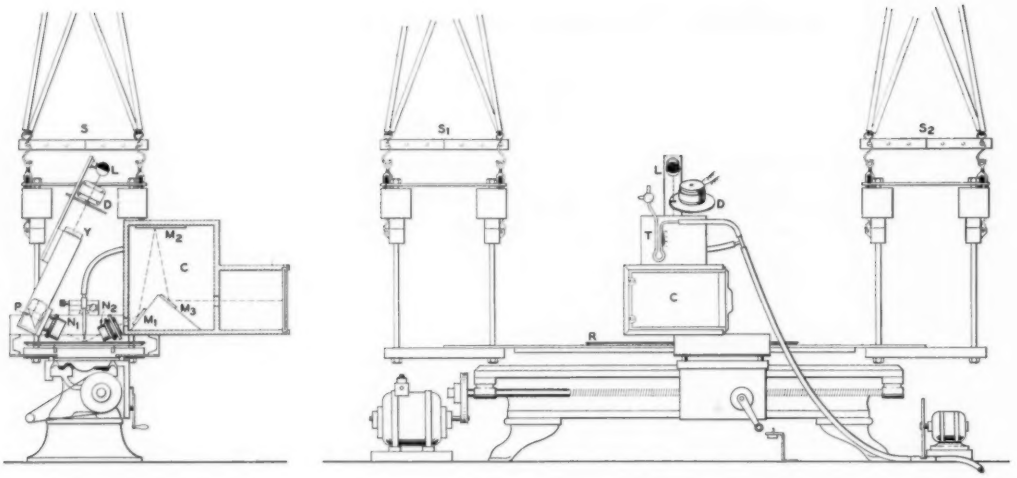
The apparatus for moving the interrupted air jets with respect to the mercury surface is shown diagrammatically in Fig. 1. In its main features it is the same as that used in the previous experiment; and shown with full descriptive legends in Fig. 1 of the previous paper. Only those elements which are different are given descriptive letters here. New features consist in a considerably longer mercury tray R; the support of the tray on two suspensions S_1 and S_2, in place of a single one; the addition of a manometer T for accurate control of the air pressure to the jets. The changes permitting the new mode of observation of the waves consist of the lamp with a linear filament L, a cylindrical lens Y to lengthen the image, a prism P to direct the light to the mercury surface through the lens N_1: and the lens N_2, which forms an enlarged image of the filament image on the mercury, through the long optical path of the camera C, with its three first surface mirrors M_1, M_2, M_3. A disk D with a narrow slot is mounted in front of the lamp L to permit stroboscopic observation of the waves when desired. The magnification of the image is very closely tenfold, so that quite small amplitudes give images large enough to study. The restriction to small amplitudes is important, for, as found in the previous study, high air pressures result in vibrations of the jet and other parts which introduce frequencies different from the fundamental one provided by the disk interrupting the air stream, the waves corresponding to those other frequencies traveling at different speeds, and complicating the phenomena.

In all cases except where noted, the frequency of the air jet was 30 cycles per second, produced by a thick, rigid circular disk mounted off center so as to produce a sine wave. The relative velocity of jet and mercury surface v was in all cases 11.28 cm/sec. The velocity of the 30-cycle waves, c, as determined from measurement of the wavelength of standing waves from a pair of jets, was 19.20 cm/sec. so that $v/c=0.585$. Reflections from the edges of the tray were prevented by filling the meniscus with a mixture of strained honey and water, water being added to replace losses by evaporation whenever observation indicated that the mixture had become too viscous. Some special features of the apparatus, used for special problems, will be described in their appropriate places.

OBSERVATIONS WITH A SINGLE JET

In Fig. 2 are shown photographs of the wave profiles for (a) a stationary medium and (b) the medium moving to the right, both taken with the rotating disk in place before the light source.

[1] Herbert E. Ives and G. R. Stilwell, J. Opt. Soc. Am. 31, 14 (1941).



R	—long mercury tray.	N_2	—lens to form enlarged image of mercury surface.
S_1, S_2	—double antivibration suspension.	C	—camera for photographing profile of waves on mercury.
L	—lamp, with straight linear filament.	M_1, M_2, M	—first surface mirrors for lengthening light path to secure large magnification in compact space.
Y	—cylindrical lens to lengthen image of filament on mercury.	D_1	—slotted disk on synchronous motor for stroboscopic observation of waves.
P	—prism for reflecting light at steep incidence to mercury surface.	T	—manometer.
N_1	—lens to image filament on surface of mercury.		

FIG. 1. Apparatus used for photographing profiles of waves on mercury surface. Letters indicate features differing from apparatus as used in previous investigation.

These exhibit the degradation of amplitude with distance from the jet, which is at the center of the photographs, and the Doppler decrease of wave-length to one side and increase on the other which are elementary phenomena of wave propagation. In order to investigate the amplitudes the rotating disk at the lamp was removed, so that the photographs would record the summation of many waves, and the photographs shown in Fig. 3a and b were obtained for the same conditions as in Fig. 2. The outstanding feature shown by these pictures is that *the amplitudes remain the same on the two sides of the jet* in spite of the very great relative velocity of jet and mercury ($v/c=0.585$) thus confirming the conclusion of the earlier paper.

This invariance of amplitude may be ascribed to two effects which offset each other. We have first the attenuation due to the expanding radius of the circular waves from their point of origin in the medium, and second the compression or expansion of the waves depending on their wave-lengths. The attenuation can be obtained for the stationary condition by measurements of the photograph 3a, or, as was done, by projecting the image to large size and measuring it directly. The results of such a series of measurements are shown in Fig. 4. In this figure the attenuation

FIG. 2. Stroboscopic photographs of wave profile fore and aft of single jet: (a) medium stationary; (b) medium moving to right.

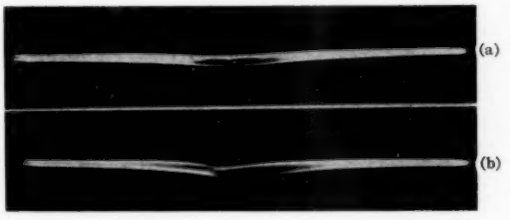


FIG. 3. Amplitudes fore and aft of single jet: (a) medium stationary; (b) medium moving to right.

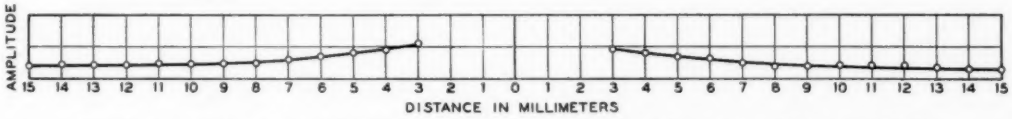


FIG. 4. Attenuation of waves from a single jet. Abscissae, distances from jet; ordinates, amplitudes.

curve is plotted according to the relation

$$x=j/d, \tag{1}$$

which is shown to represent accurately the data. The amplitude is inversely as the distance from the point of impact of the air jet.

When the medium is in motion the center of any given circular wave has moved away from the jet by the distance vt, where t is the time from the initiation of the wave. The amplitude of any wave at a distance d from the jet is therefore,

$$a'=J/d(1\pm v/c). \tag{2}$$

Because of the motion of the medium alone, we should expect the amplitudes to be increased on the side toward which the medium is moving, and vice versa.

On the other hand when the waves are compressed or expanded, due to the Doppler effect, if the same amount of medium is to be included in the wave, its amplitude must increase or decrease in inverse proportion to its wavelength, or by a fraction $(1\pm v/c)$. This factor exactly compensates the dissymmetry of attenuation introduced by motion, in agreement with the observed result.

PHENOMENA WITH TWO JETS IN THE LINE OF MOTION OF THE MEDIUM

With two jets located in the line of motion of the medium we observe the occurrence of standing waves. At the mid-point between the jets the amplitudes are equal and in phase, giving a loop; the neighboring nodes are stationary and of "zero" amplitude. Nearer either jet the amplitudes from the two jets are different, resulting in a broadening of the nodes. These phenomena are shown in Fig. 5a. When the medium is in motion the amplitudes due to the two jets remain the same as for the stationary condition, as shown in the last section, consequently the amplitudes of the nodes and loops of the standing wave system also duplicate the stationary conditions. This is shown by Fig. 5b.

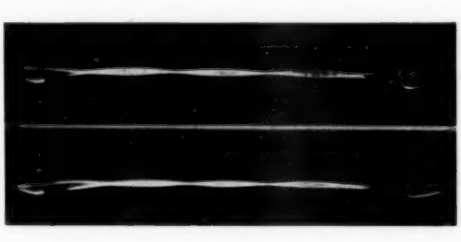


FIG. 5. Standing waves between two jets: (a) medium stationary; (b) medium moving to left.

There is, however, a difference in the positions and lengths of the standing waves. While we are here primarily concerned with the question of amplitudes it is of some interest to look into these alterations of position and length, particularly as influenced by the properties of the medium.

The formation of standing waves between two synchronous sources of continuous wave trains may be conveniently studied by means of the graphical construction of Fig. 6. In Fig. 6a let the abscissae represent distances measured in the line joining the sources, the ordinates time. The paths of light pulses from the two sources are then represented in time and position by the diagonal lines $(s/t=c)$. The crossing points of these diagonal lines are points of like phase of intersecting pulses. The lines joining these intersections (shown dashed) are the loci in space of the standing wave loops.

Let us now construct a similar diagram (Fig. 6b) for the condition with the medium moving to the left, with some uniform velocity v; the velocities of the waves, with respect to the sources are then $c+v$ and $c-v$, represented by diagonal lines of different slope, as shown, the value of v/c in the diagram being 0.4. The intersections of these diagonal lines give us a new set of standing waves, as shown by the dashed lines, shifted to the left, and closer together than in Fig. 6a.

Consider first the shift in position. This could

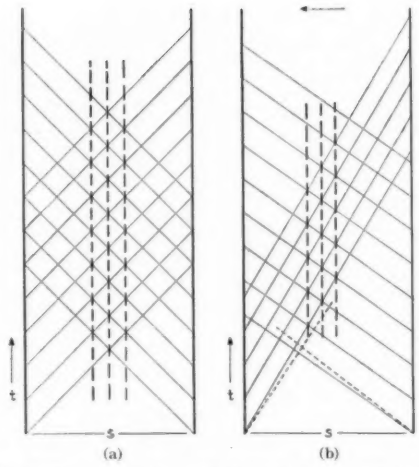


FIG. 6. Diagrammatic analysis of formation of standing waves between two sources: (a) medium stationary; (b) medium moving to left.

be offset by starting all the waves from the source to the right a short time later. Now in the actual experiment of starting two sets of waves at different places at the same instant it would be necessary to establish the simultaneity of the pulses. To do this recourse would have to be made to clocks, one at one source (say that at the left) the other, set with the first, transported to the second source (at the right). In this operation the transported clock would be retarded by its change of rate in motion. It can be shown that this change of setting of a clock moved (infinitely slowly) from the left source to the right is in fact just the amount by which the waves from the source to the right should be delayed in starting, in order to restore the symmetry of the pattern. Or "simultaneity" of emission could be established by the equivalent procedure of sending a light signal and ascribing to it the invariant velocity c.

Next consider the distances apart of the standing waves. In Fig. 7 is shown on enlarged scale one of the small parallelograms formed by the intersecting wave crests in Fig. 6(b). In it the distance between standing waves is indicated by the horizontal line d; the interval between waves by $\tau=\tau_1+\tau_2$. Now from the construction

$$\begin{aligned} d/\tau_1&=c+v \\ d/\tau_2&=c-v \end{aligned} \qquad (3)$$

so that

$$(\tau_1+\tau_2)/d=\tau/d=1/(c+v)+1/(c-v)$$

or

$$d=\frac{\tau c}{2}\left(1-\frac{v^2}{c^2}\right)=\frac{\lambda_s}{2}\left(1-\frac{v^2}{c^2}\right), \qquad (4)$$

where λ_s is the length of the waves emitted by the stationary sources. The standing waves are shortened, in qualitative agreement with the photograph Fig. 5b.

This effect of motion is eliminated if (1) we reduce the frequency of the light sources by the factor

$$(1-v^2/c^2)^{\frac{1}{2}}$$

and (2) if we diminish the separation of the sources by the same factor, and at the same time measure the lengths of the standing waves by a scale similarly shortened. These contractions are those required by an ideal medium in which the velocity is independent of wave-length. For our experimental case, where $v/c=0.535$, the factor $(1-v^2/c^2)=0.658$; actual measurement of the photographs show the ratio of wave-lengths between Fig. 5b and Fig. 5a to be 0.852. The explanation of this difference is to be found in the fact that in capillary waves in material liquids the velocity depends upon the wave-length; the shorter waves from the source to the left in Fig. 6b move faster, the longer waves from the source at the right move slower, as indicated by the dotted lines, whereby the oblique parallelogram formed by the wave crests are rotated part way back toward the shape for

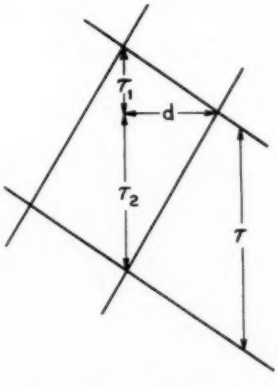


FIG. 7. Derivation of distances between standing waves in a moving medium.

the stationary condition (Fig. 5a). In a medium having surface tension the Fitzgerald contraction and the Larmor-Lorentz frequency change overcompensate for the effects of motion.

STANDING WAVES FROM A REFLECTOR

Some experimental difficulties are presented by the problem of moving a reflecting edge along a mercury surface, since if the edge has any considerable depth it will move the mercury bodily, setting up surges and waves sufficient to mask the wave phenomena. Since however, the behavior of a moving mirror has figured frequently in discussions of wave phenomena it was considered highly desirable to devise, if possible, a reflector which could be moved without introducing disturbing effects. Various forms of reflectors were investigated, resulting finally in one which could be moved at the desired speed along the mercury surfaces without stirring it up. The "mirror" is shown diagrammatically in Fig. 8. It consists of a section of glass rod of small diameter, to the two ends of which are attached long very "springy" thin glass threads, just stiff enough to hold the rod normal to the direction of motion, but not to push the rod downward below its normal floating position. The threads were attached to the moving carriage and the rod riding on the mercury surface assumed a floating depth just sufficient to stop and reflect the capillary waves. The photograph, Fig. 9, shows the mirror in position as actually used.

The standing wave system formed by this

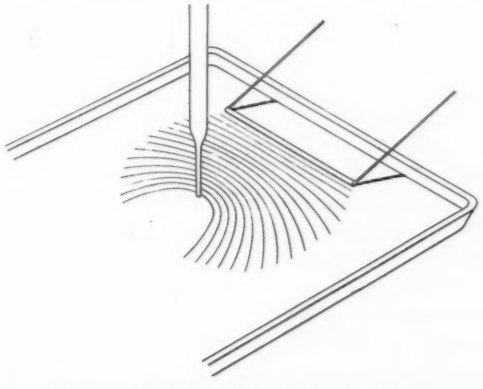


FIG. 8. "Mirror" used for study of standing waves by reflection.

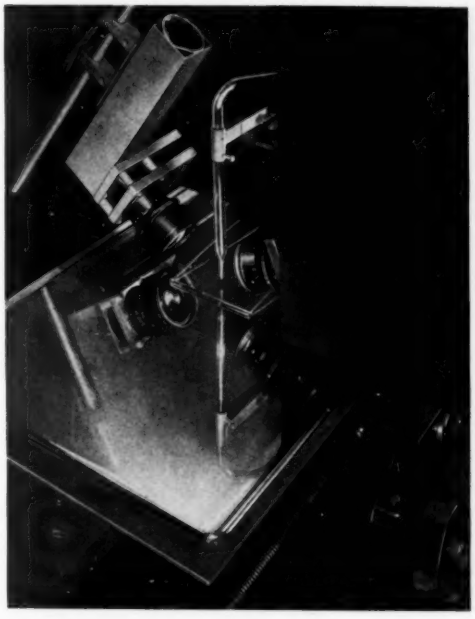


FIG. 9. Photograph of jet, "mirror," and optical system.

mirror is shown in Fig. 10, where in order to exhibit clearly the whole system of waves from source to reflector a high frequency (90 cycles per second) was used. The occurrence of a loop at the mirror is shown to the right, the first node from the mirror, where incident and reflected amplitudes are substantially equal, is of negligible amplitude, while the nodes nearer the source, to the left, where the amplitudes of the meeting waves are unequal, are less well differentiated from the loops. To the right of the mirror the waves are absent.

In Fig. 11a is shown a photograph of the standing waves for the stationary condition for the frequency of 30 cycles, and in Fig. 11b the standing waves with the system moving. In both photographs the mirror is outside the field to the right. It is clear that with the source-mirror system we have the same performance as with the system of two sources, that is the amplitudes of loops and nodes are uneffected by motion.

That this is to be expected is made clear by considering the situation at the midpoint between two real sources and at the surface of the

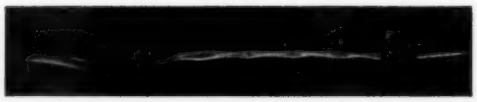


FIG. 10. Profile of standing waves formed by mirror.

mirror. Let us first make a graphical construction with the medium moving to the left past the fixed source and mirror. This (Fig. 12) has been drawn for the case of $v/c=0.4$ and with the tenth wave from the source just tangent to the mirror. For each reflected wave we calculate the virtual point of origin by the relation

$$D_2=(R_2^2-r^2)^{\frac{1}{2}}, \tag{5}$$

where using the symbols of the next Fig. (13), r is the distance from the axis at which the wave cuts the mirror, and

$$R_2=D_2+n\lambda(1+v/c), \tag{6}$$

in which n is the number of the wave as counted to the left from the mirror. The intercept r is computed from the relation

$$(m\lambda)^2-(D+m\lambda v/c)^2=r^2, \tag{7}$$

where m is the number of the wave counted from the source.

Inspection of this figure shows that (1) the virtual sources of the reflected waves are closer to the mirror than the real source, (2) the virtual sources for successive reflected waves are displaced in the direction of motion of the medium.

The relations shown graphically in Fig. 12 may now be made quantitative by solving Eqs. (5), (6), and (7). From Fig. 13 we have

$$\begin{aligned} R_1&=D_1+n\lambda(1-v/c) \\ R_2&=D_2+n\lambda(1+v/c) \end{aligned} \tag{8}$$

and

$$\begin{aligned} R_1^2&=D_1^2+r^2 \\ R_2^2&=D_2^2+r^2. \end{aligned} \tag{9}$$

Solving these relations we get

$$D_2=\frac{D_1(1-v/c)-2n\lambda v/c}{(1+v/c)}. \tag{10}$$

For a wave just tangent to the mirror, for which $n=0$

$$D_2=D_1\frac{(1-v/c)}{(1+v/c)}. \tag{11}$$

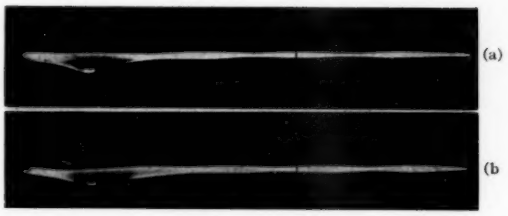


FIG. 11. Standing waves from a mirror: (a) medium stationary; (b) medium moving.

Now consider two successive waves which leave the mirror; call the distances of their virtual sources D_2 and D_2^1 and the distances of the virtual sources of these waves before reflection D_1 and D_1^1. We then have

$$D_2^1=\frac{D_1^1(1-v/c)-2(n+1)\lambda v/c}{(1+v/c)} \tag{12}$$

and the displacement of the virtual source in the mirror for successive waves

$$D_2^1-D_2=\frac{(D_1^1-D_1)(1-v/c)-2\lambda v/c}{1+v/c}. \tag{13}$$

We evaluate $D_1^1-D_1$ from the relations

$$\begin{aligned} D_1&=D+m\lambda v/c \\ D_1^1&=D+(m+1)\lambda v/c, \end{aligned}$$

giving

$$D_1^1-D_1=\lambda v/c, \tag{14}$$

getting finally

$$D_2^1-D_2=\frac{\lambda v}{c}\frac{[1-v/c-2]}{1+v/c}=-\frac{\lambda v}{c}. \tag{15}$$

Inspection of these results (Eqs. (11) and (15)) shows at once that our graphical representation, Fig. 12, exactly reproduces the construction used in exhibiting the wave phenomena between two real sources with a moving medium. Since for real sources we have found the amplitudes of the waves at all points unaffected by motion of the medium, this conclusion follows at once for the case of the mirror.

The conclusion that the amplitudes of incident and reflected waves at the surface of a moving mirror are identical was *assumed* by Larmor[2] in

[2] *Encyclopedia Britannica*, ninth and tenth editions, article, "Radiation." International Congress of Mathematics, Cambridge (1912), p. 208.

his pioneer study of the pressure of radiation on a moving mirror. It was however sharply contested by Edser[3] who showed that for electromagnetic waves there must occur a change of amplitude on reflection,—a claim that appears to have been conceded by Larmor.[4] Edser based his proof of the formula for radiation pressure on the contention that the energy density in the medium was a function solely of amplitude, frequency playing no part, which Larmor could not accept.[4] We find also that Abraham[5] and Lorentz[6] state categorically that the amplitude of the reflected radiation is changed in the same ratio as the frequency. In the light of our discussion it appears that the correct statement is:

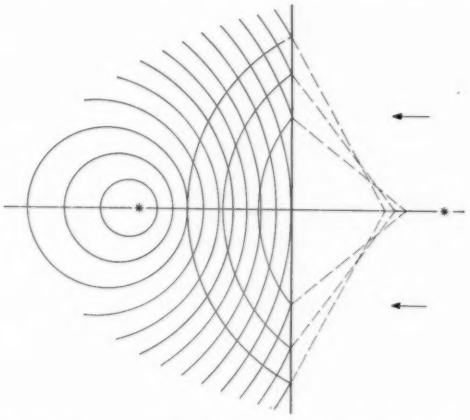


FIG. 12. Graphical representation of reflection of waves from moving mirror.

[3] E. Edser, Phil. Mag. **28**, 508 (1914).
[4] J. Larmor, Phil. Mag. **28**, 702 (1914).
[5] M. Abraham, *Theorie der Elektricitat* (Leipzig, Teubner, 1905), p. 335–6.
[6] H. A. Lorentz, *The Theory of Electrons* (New York, Stechert, 1923), p. 62. "As to the amplitude, it is changed in exactly the same ratio as the frequency, so that the reflected intensity is diminished by a motion in one direction and increased by a motion in the other direction."

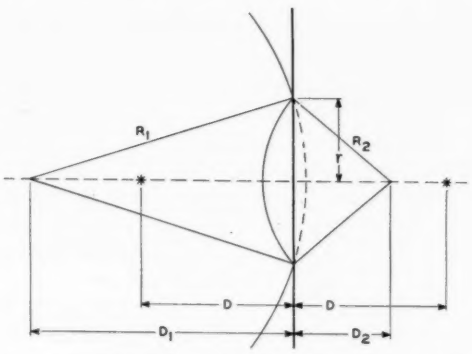


FIG. 13. Derivation of position of virtual source of waves leaving surface of moving mirror.

The amplitudes are changed in the ratio of the frequencies, but the effect of this change is exactly offset by the attenuation consequent on the distance assumed by the virtual source created by the moving mirror. If reflected and incident amplitudes were actually different, in a co-moving system of source and mirror, as an uncritical acceptance of the above authors' statements would indicate, *the nodes of the standing waves would vary in amplitude with the velocity of the system*, and would thus furnish a method of detecting absolute motion.

SUMMARY

Experimental apparatus and methods have been developed by which the *profiles* of ripples on a mercury surface can be photographed, with the surface in motion relative to the source or to a mirror. It is found that in all cases the *amplitudes* of vibration remain invariant with motion of the medium. The factors from which this behavior derives are analyzed, and the effects of the Fitzgerald and Larmor-Lorentz contractions are discussed.

Pressure of Radiation in Moving System

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(Received October 17, 1941)

THE results of the investigation described in the preceding paper[1] make possible a complete discussion of the role of radiation pressure in radiation measurements in a moving system. Various aspects of this problem have been treated by Larmor,[2] Levi-Civita,[2] Poynting,[3] Bucherer,[4] H. A. Lorentz,[5] Abraham,[6] and others, but their results have not been in complete agreement. In particular the fact that (as will be shown) both irradiation and radiation pressure are unaffected by motion of the entire system of source and receiver was not established by those discussions, which in fact actually call for first-order variations of radiation pressure with motion.

We start this discussion, with Larmor,[2] by studying the pressure of radiation from a point source on a moving perfect reflector. We have found, from the preceding paper, that the amplitude of vibration in the medium in front of the mirror is the same for both incident and reflected radiation. The energy density in incident and reflected wave trains is then proportioned to the square of the frequencies.[2] For the case of the mirror moving toward the source, if E_i is the energy density in the incident radiation E_r that in the reflected, we have

$$E_r = E_i\left(\frac{c+v}{c-v}\right)^2. \tag{1}$$

We now set down the equation expressing the fact that the work done by moving the mirror with velocity v against the radiation pressure P is equal to the net rate of flow of energy into the mirror.[5]

$$E_r(c-v) - E_i(c+v) = Pv. \tag{2}$$

Substituting from (1) we have

$$E_i\frac{(c+v)^2}{(c-v)^2}(c-v) - E_i(c+v) = Pv$$

or

$$P = 2E_i(c+v)/(c-v). \tag{3}$$

Let us now, for subsequent use, consider the pressure from the standpoint of the momentum of radiation. We start with the definition of pressure as the quotient of the change of momentum by the time in which the change occurs, or, as alternatively expressed, that the impulse (pressure×time) is equal to the change of momentum. We must, in order to do this, ascribe to the radiation (of velocity c) a certain mass; call this mE per unit volume. Consider a train of radiation of energy density E_i emitted from a source in time Δt; its mass equivalent will be

$$mE_i c\Delta t$$

and its momentum

$$mE_i c\cdot\Delta t\cdot c = mE_i c^2\Delta t.$$

After reflection the length of the train will be $[(c-v)/(c+v)]\cdot c\cdot\Delta t$, its energy density will be $E_i(c+v)^2/(c-v)^2$ so that its momentum will be $-mE_i[(c+v)/(c-v)]c^2\Delta t$.

The change of momentum

$$mE_i c^2\left(1+\frac{c+v}{c-v}\right)\Delta t$$

takes place in a time $c/(c+v)\Delta t$, so that we have

$$mE_i c^2\left(\frac{2c}{c-v}\right)\Delta t = \frac{Pc\Delta t}{c+v} = \left[2E_i\left(\frac{c+v}{c-v}\right)\Delta t\Big/(c+v)\right],$$

from which

$$m = 1/c^2,$$

so that we derive the *mass equivalent* of 1 cc of radiant energy of density E to be

$$E/c^2. \tag{4}$$

We next investigate the pressure on a completely *absorbing* surface, moving toward the point source with velocity v. This we do by considering the change of momentum of radiation,

[1] H. E. Ives and G. R. Stilwell, J. Opt. Soc. Am. **32**, 25 (1942).
[2] "On the Dynamics of Radiation," Int. Congress of Mathematics (Cambridge, 1912), p. 208, and references to earlier articles.
[3] J. H. Poynting, "Radiation in the Solar System," Phil. Trans. **A202**, 525–552 (1940).
[4] A. H. Bucherer, Ann. d. Physik **11**, 274–276 (1903).
[5] H. A. Lorentz, Proc. Amst. Akad. **4**, 678 (1940).
[6] M. Abraham, *Theorie der Electricität* (Teubner, Leipzig, 1905), Vol. II, p. 384.

by using the mass equivalent just obtained. Taking as before the train of radiation emitted in time Δt from the source, we have for the impulse equation:

$$\frac{E_i c\Delta t}{c^2}\cdot c+\frac{(E_A t)}{c^2}\cdot v=Pt, \tag{5}$$

where E_A is the rate of absorption of energy by the surface, and t is the time of incidence of the wave train, so that $E_A t/c^2$ is the mass equivalent of the absorbed radiation, which moves with velocity v.

Now in order to obtain E_A we use the relation

$$E_i(c+v)+Pv=E_A, \tag{6}$$

which states that the absorbed energy comes both from the stream of radiation and from the work done in moving the surface.[5] For t we have $c\Delta t/(c+v)$ seconds. Making these substitutions we have

$$\frac{E_i c\Delta t}{c^2}\cdot c+\left(\frac{E_i(c+v)+Pv}{c^2}\right)\left(\frac{vc\,\Delta t}{c+v}\right)=\frac{Pc\Delta t}{c+v},$$

from which

$$P=E_i(c+v)/(c-v) \tag{7}$$

or the pressure on a completely absorbing surface is *exactly half that on a perfect reflector moving with the same velocity.*

This value for the pressure on an absorbing surface is significantly different from ones heretofore published. Lorentz puts the pressure simply equal to the energy density, or $P=E_i$; Poynting and Abraham make it $P=E_i(1+v/c)$.[7] *Either of these values demands that radiation pressure shall be different in a system of co-moving source and absorber from its value in a stationary system.* This was pointed out by Abraham,[6] who contented himself with the remark that, because of the minuteness of radiation pressure, the effect (although of the first order in v/c) would be practically impossible to detect.

The invariance of irradiation and radiation pressure in a moving system which results from the value of radiation pressure here derived may be illustrated by the single example of co-moving (point) source and absorbing surface. For the case of the system stationary we have simply that

$$P_s=E_s, \tag{8}$$

$$R_s=E_s c, \tag{9}$$

where R_s is the irradiation, E_s the energy density for the stationary source. For co-moving source and absorber moving in the direction opposite to the radiation, we have

$$R_M=E_M(c+v)+P_M v, \tag{10}$$

$$P_M=E_M(c+v)/(c-v). \tag{11}$$

In order to solve these relations we need E_M, the energy density due to the moving source. Since the amplitude at the absorbing surface is the same whether the system is moving or stationary,[8] the energy density will be proportional to the square of the frequency or

$$E_M=\frac{E_s}{(1+v/c)^2}.$$

We now however must introduce the Larmor-Lorentz change of frequency with motion, $\nu=\nu_0(1-v^2/c^2)^{\frac{1}{2}}$, giving finally[9]

$$E_M=E_s\frac{(1-v^2/c^2)}{(1+v/c)^2}=E_s\left(\frac{c-v}{c+v}\right), \tag{12}$$

[7] Larmor, in an appendix to Poynting's paper "On the momentum of radiation," as reproduced in the latter's *Collected Works*, points out (p. 433) that Poynting's expression for the pressure of a wave-train against a traveling obstacle is in agreement with his own expression for the pressure on a moving mirror, since

$$E_i\left(1+\frac{v}{c}\right)+E_r\left(1-\frac{v}{c}\right)=2E_i\left(\frac{c+v}{c-v}\right)=P.$$

This does not however prove Poynting's value correct since *also*

$$E_i\left(\frac{c+v}{c-v}\right)+E_r\left(\frac{c-v}{c+v}\right)=2E_i\left(\frac{c+v}{c-v}\right),$$

the value of $E_r=E_i(c+v)^2/(c-v)^2$ being used in both cases. Poynting's expression calls for the incident and reflected radiations to contribute different pressures to the mirror, the expression (7) makes the contributions alike.

[8] The reduction of the distance r to $r'=r(1-v^2/c^2)^{\frac{1}{2}}$ by the Fitzgerald contraction is offset by the Larmor-Lorentz change of frequency and consequent reduction of amplitude, so that this relation holds equally for r and r'.

[9] In the similar electrodynamic case, for a Hertzian oscillator, as solved by the use of retarded potentials (Bucherer, Ann. d. Physik **11**, 276 (1903)) the energy density is

$$E=\frac{\bar{K}a^2n^4}{8\pi r^2c^3(1+v/c)^2}.$$

Taking into account the length and frequency contractions we put $n=n_s(1-v^2/c^2)^{\frac{1}{2}}$, and $r=r_s(1-v^2/c^2)^{\frac{1}{2}}$, and get

$$E=\frac{\bar{K}a^2n_s^4(1-v/c)}{8\pi r_s^2c^3(1+v/c)}=E_s\left(\frac{1-v/c}{1+v/c}\right),$$

in agreement with our solution (12) derived from consideration of mechanical wave motion.

from which

$$P_M = E_S\left(\frac{c+v}{c-v}\right)\left(\frac{c-v}{c+v}\right) = E_s = P_s \qquad (13)$$

and

$$\begin{aligned} R_M &= E_s(c-v)+P_s v \\ &= E_s(c-v)+E_s v \\ &= E_s c = R_s. \end{aligned} \qquad (14)$$

Equations (13) and (14) compared with Eqs. (8) and (9), prove the invariance of the quantities with motion of the system. (For the case of a co-moving source and perfect reflector this invariance with motion follows at once from (3) and (12).)

The derivation of the correct expression (7) for radiation pressure on a moving absorbing surface thus makes possible demonstration of the invariance of irradiation and radiation pressure in moving systems. The invariance holds to the first order of v/c without the introduction of the Fitzgerald-Larmor-Lorentz contractions, and holds to all orders when these are invoked, thus placing irradiation and radiation pressure in the same category as the observed arrival times and places of light signals. This investigation confirms the result of H. A. Lorentz, in 1903,[9] as to invariance of irradiation in a system of co-moving source and receiver to the first order of v/c, extends it to all orders, and eliminates the variation of radiation pressure with velocity which was present in his solution.

Letter to the Editor

A Low Brightness Meter

A. H. TAYLOR
Lighting Research Laboratory, General Electric Company, Nela Park, Cleveland
December 29, 1941

IN re-reading the manuscript of my article on this subject, published in the December issue of the Journal, it occurs to me that one sentence of the article might convey a meaning not intended. The sentence is: "It is customary to assume that the unit of brightness has a fixed value in terms of energy-density of the stimulus regardless of the spectral distribution of the radiant energy from the light-source." The meaning intended would have been clearer with the following substitution for the above statement: "It is customary to assume that the apparent brightness of an object is *directly proportional* to the flux density of the reflected or emitted flux. Thus, if the flux density is increased by a factor of 100 or 1000, the brightness is also increased by the same factor."

Some of the factors involved in photopic and scotopic vision are discussed in greater detail in an article "Brightness and Brightness Meters" to be published in *Illuminating Engineering* in January, 1942. Because of the Purkinje effect, the unit of brightness becomes a flexible unit in terms of flux density for brightnesses appreciably below one footlambert.

The magnitude of the Purkinje effect may be indicated by some computed values, in which the scotopic luminosity curve of Weaver[1] at 6.5×10^{-5} footlambert has been used. If we assume the brightness to be directly proportional to energy-density for an equal energy spectrum, relative brightnesses for other sources reduced in energy densities in the same proportion would be as follows:

		Relative Brightnesses
Equal energy	1.0 Fl.	6.5×10^{-5} Fl.
6500° daylight	1.0	7.1×10^{-5}
2400° black-body	1.0	3.67×10^{-5}
Sodium	1.0	0.9×10^{-5}

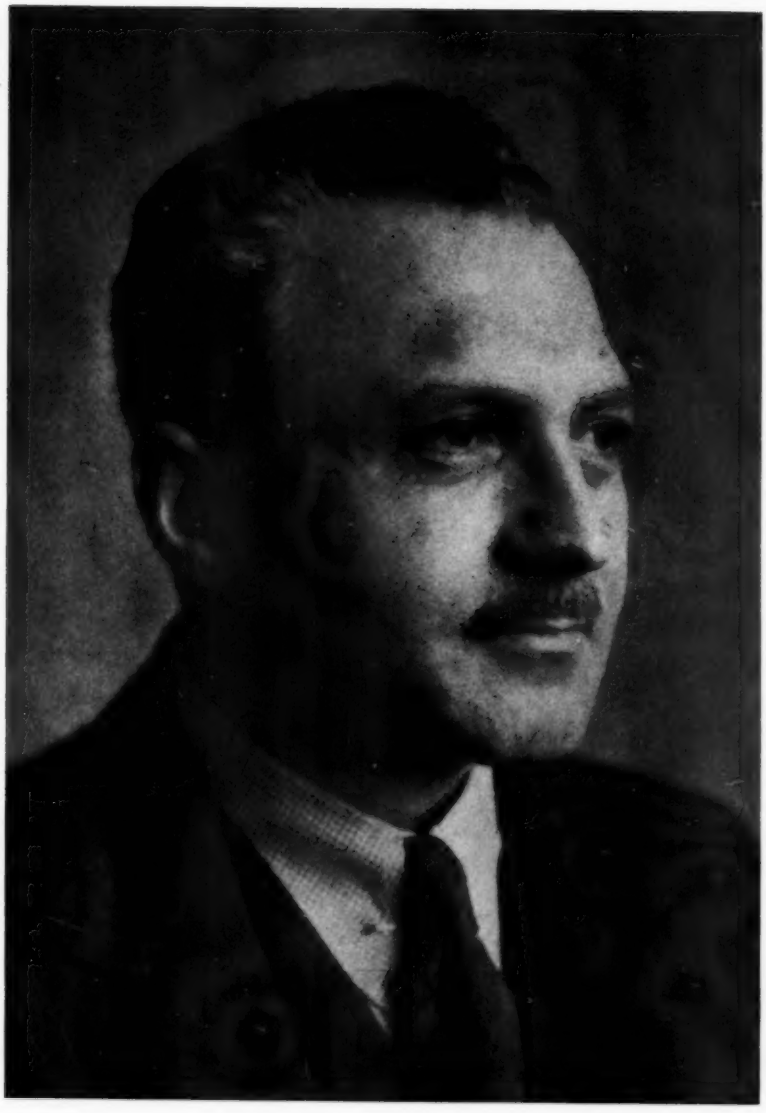
From the above it is obvious that the scale for low brightnesses is dependent upon the light-source used in calibrating the meter, if it is assumed that the brightness is directly proportional to the light-flux density. Our practice in basing our scale on light from a tungsten-filament lamp at approximately 2400°K is in conformity with British practice, hence our scale is consistent with theirs.

[1] K. S. Weaver, J. Opt. Soc. Am. **27**, 36 (1937).

The Frederic Ives Medal for 1941

Awarded by The Optical Society of America to Selig Hecht,
Professor of Biophysics at Columbia University,
for distinguished work in optics.

This medal, awarded biennially, was endowed in 1928 in honor of Frederic Ives, distinguished for his pioneer contributions to color photography, photoengraving, three-color process printing, and other branches of applied optics.



SELIG HECHT

Doctor Selig Hecht

IVES MEDALIST FOR 1941

AT its New York meeting in October, 1941, the Optical Society of America awarded the Frederic Ives Medal to Dr. Selig Hecht, Professor of Biophysics at Columbia University. This medal is awarded biennially for distinguished work in optics, and its past recipients have been as follows:

In 1929, Edward Leamington Nichols;
In 1931, Theodore Lyman;
In 1933, Robert Williams Wood;
In 1935, George Ellery Hale;
In 1937, Herbert Eugene Ives;
In 1939, August Herman Pfund.

The presentation of the medal to Dr. Hecht took place at the Annual Dinner of the Society, which was held in the Hotel Pennsylvania, with more than a hundred and fifty members and guests of the Society attending.

Presenting Dr. Hecht, the chairman of the committee appointed to nominate a candidate for the award, spoke as follows:

"The other day I was listening to two scientists talking. In the course of conversation, one asked the other if he knew anything about the sensitivity of birds' and wild animals' eyes to the ultraviolet and infra-red regions of the spectrum. 'No,' said the other, 'but if you really want to know, ask Hecht. He knows everything about vision.'

"A quick glance through Professor Hecht's 90-odd published papers will certainly bear out that statement. It would be hard to find any property of the sense organs of vision, and of the means by which nerve impulses are initiated by light, that is not discussed in one or another of his papers.

"Hecht's approach to any problem is essentially critical and historical. If he detects what appears to be a weakness in a theory, he will study the literature in great detail, going back 100 years if necessary. He will analyze all available data, study any discrepancies, and then undertake a careful redetermination of the data. In this way he has often shown that a discrepancy was more apparent than real, or else he has found out some entirely new fact that helps us to drop into place one more piece of that overwhelming jigsaw puzzle known as 'Physiological Optics.'

"In one of his papers (73), he opens with the following remark: 'One of the satisfactions in applying a theory comes from its unexpected illumination of data which, though well known, have long remained obscure in their interpretation and unrelated to the rest of the field.' This has been particularly true in Hecht's work and in that of his many students and disciples.

"In spite of his voluminous writing, Hecht's avowed aim is always to simplify. In a paper published in the Journal of this Society in 1924 (20) he makes the following statement: 'Anyone acquainted with the field of physiological optics knows that there exists a staggering mass of literature describing different phases of the subject, . . . (and) it is, therefore, with much trepidation that one presumes to write a paper concerned with the phenomena of vision. My hope, however, in doing so, is not to add to the existing material, but rather to subtract from it. If one can succeed in bringing a number of scattered phenomena into orderly arrangement, one decreases by so much the welter of unrelated facts that constitutes the present literature of visual science.' He then goes on to urge the would-be investigator in this field to 'cease spinning webs of shimmering words, and turn his energies to the painstaking investigation of visual phenomena under conditions calculated to answer definite and simple questions.'

"Selig Hecht was brought to this country from Austria by his parents as a child, in the 1890's. He obtained his Bachelor of Science degree at the College of the City of New York in 1913. His first appointment was that of pharmacologist in the Bureau of Chemistry of the U. S. Department of Agriculture during 1913–14, after which he moved to Harvard where he obtained his Ph.D. in 1917 under G. H. Parker, working on sensory physiology. He held the position of Assistant Professor of Physiology in the College of Medicine, Creighton University, Omaha, Nebraska from 1917–1921. He then became a National Research Council Fellow in Physical Chemistry

and Physiology from 1921–1926, during which period he not only worked at Harvard, but also abroad, at Liverpool University in England, and at the Zoological Station in Naples, Italy. In 1926 came his final appointment, an Associate Professorship of Biophysics at Columbia University, which was raised to a full professorship two years later.

"While working in the Marine Biological Laboratory, Woods Hole, Massachusetts, Dr. Hecht was attracted to the problem of measuring the light sensitivity of the clam, Mya, and certain other related animals. He found that Mya acts like a regular photochemical experiment. On exposure to light, it has a definite induction period of two or three seconds, after which the reciprocity law of Bunsen and Roscoe is faithfully satisfied. This remarkable behavior was an obvious challenge to anyone such as Hecht interested in both photochemistry and biology; and if we add to these interests, the ability to handle moderately difficult mathematics with ease, together with a good knowledge of physical chemistry, we find that his qualifications and training were ideal for his chosen field of study.

"From the photosensitivity of 'Mya,' it was but a short stretch to the human retina, which is first and foremost a photochemical organ. It was early clear to Hecht that in the retina there must exist a substance, or substances, which absorb light, liberating other substances in the process. Moreover, there must be a continuous supply of this photosensitive material, or else vision would soon come to an end, and the whole process must do something to excite electrical nerve currents which convey the light signal to the brain. In one of his papers (75), Hecht goes on to say that 'the ultimate place of origin of the impulses passing up the optic tracts is in the action of light on the sensory elements. Therefore, the various properties of vision and photoreception must owe some of their quantitative form to the characteristics of the reactions which take place between light and the sensitive elements. It is obviously of first-rate biological interest to discover how close this correlation is.' Hecht has at times been criticized for devoting so much attention to the photochemistry of the retina, but in his view, the photochemical reactions going on there are probably the seat of all the phenomena of vision, since there the light waves are absorbed and transformed in some way into nerve impulses.

"Subjects of immediate interest in his study of the photochemical theory of vision were, of course, light adaptation and the related topics of the threshold of vision, the relation between visual acuity and illumination, intermittent stimulation by light, and the phenomena of flicker. These matters have occupied Hecht's attention very deeply during the succeeding years since 1918. He tackled color vision with his usual vigor, and I well remember the impression made upon my mind by his energetic presentation at Ithaca in 1929, of his paper (41), 'The development of Thomas Young's theory of color vision.' Here Hecht was deliberately iconoclastic, and showed a complete independence of thought. That this paper should bring forth an immediate rebuttal from other color workers was to be expected, and it certainly served to make us think more deeply about the so-called 'fundamentals' of the trichromatic theory of color vision.

"From color vision we naturally think of color blindness, and in this field too Hecht has contributed his share of significant, careful observations and his usual searching analyses. For example, he early realized that to the color-blind person different colors become apparent as differences in saturation, and that at one point in the wave-length scale of a dichromat his sensation curves intersect and form white (68). To either side of this point, colors appear very unsaturated, the saturation increasing as the wave-length departs from the central point.

"Hecht is unstinting in giving credit both to his co-workers and to those scientists who preceded him in this endless quest for knowledge. It is interesting to discover in one of his papers (51, page 272) that he pays tribute to the work of the distinguished donor of the medal which we are awarding tonight. He writes: 'H. E. Ives' 'theory of intermittent vision' is the best and most concrete effort at including all the data existing at that time (1922).' However, he then proceeds in characteristic fashion to pick three large holes in Ives' theory, after which he substitutes a new theory of his own, based on the much more complete knowledge of photochemistry and neurology available at the present day.

"The mathematical approach exemplified in this paper makes us realize how many scientists are being limited and hampered in their work by an insufficiently broad and catholic training.

"For all his activity in the direct problems of human vision, Hecht has not neglected comparative studies of other animals. In 1929 he undertook the unpromising task of measuring the visual acuity of the bee (36) and the fruit fly (54). By ingenious experiments, he succeeded in doing this over wide ranges of brightness and color. Even the owl has come into its share of attention, and in a recent paper (89) Hecht has studied the frequency and intensity characteristics of the owl's visual response, using the contraction of the pupil as an indication that the radiation has been perceived.

"Hecht's latest problem is the very interesting connection between night blindness and Vitamin A (88). Here he is back on his old hunting-ground of the photochemistry of visual purple and its disintegration products. But now visual purple has graduated from an indefinite substance, which could be extracted from a retina only with great difficulty as a very weak solution, to a full-fledged protein with a molecular weight of about 280,000. Hecht's former student, Wald, has studied the disintegration products of visual purple in great detail (78), finding one of them, retinene, to be closely related to Vitamin A. This provides the necessary clue to the long-recognized connection between vision and certain deficiencies in the diet. At the present time Hecht is working energetically to apply the tests of dark adaptation to the selection of military aviators.

"Mr. President: The committee recommends unanimously that the Frederic Ives Medal for 1941 be awarded to Professor Selig Hecht, teacher, analytical thinker, and research worker, for his thorough and immensely detailed study of the phenomena of vision, and for the inspiration that he is continually giving to other brilliant workers in the field of physiological optics."

RUDOLF KINGSLAKE
Chairman, Committee on Award of the Frederic Ives Medal for 1941

The Ives Medal was then presented by President Gibson of the Society, and Dr. Hecht responded as follows:

"When Dr. Gibson first informed me of the Ives award, I supposed that when the time for the ceremony actually arrived, I would be so accustomed to the thought of the award that my emotions would not run away with me, and that I would have a properly graceful speech of acceptance ready. That was a dream; the reality of the award has moved me so that I have forgotten most of the things I was going to say. I am reduced merely to saying that I am deeply affected by the confidence which my fellow scientists have in my work, and I thank them and you for awarding me this medal. To be put in the company of Theodore Lyman, R. W. Wood, and the other previous recipients is a distinction I shall always treasure; it will help sustain me especially during those wretched hours, through which we all pass, when experiments fail and we consider ourselves the meanest of creatures.

"Approval is something we all desire. My youthful life became rosy when Jacques Loeb spoke kindly of a paper. Alas, one of the discoveries of maturity is that the number of pats on the back received decreases sharply, while the number of pats to be given to the youngsters growing up steadily increases. Perhaps one reward of oncoming age is just such public approval as you have given me tonight.

"I have long been an admirer of Herbert Ives, the founder and a previous recipient of this medal. His work has been close to mine, and it makes me particularly happy that he was on the committee which has given the medal to me.

"One out of the many forgotten thoughts does come back to me. For a dozen years I have had a research colleague and friend who has worked with me on visual problems. He is Dr. Simon Shlaer. Much of what I have done is due to his ingenuity in devising apparatus and gadgets, and to his criticism of the ideas. With the permission of the Committee of Award, I would like to think that part of the Ives Medal goes to him."

PAPERS RELATING TO VISION BY SELIG HECHT

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The Quantum Relations of Vision*

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I. MINIMUM ENERGIES

THIS evening I wish to present the essentials of an investigation recently made by Dr. Simon Shlaer, Dr. Maurice H. Pirenne and myself on the minimum energy required to produce a visual effect. In reporting this investigation I would like to lay before you more than the data and the conclusions. If possible I want to convey something of the pleasure with which this research has been associated, something of the intellectual excitement which surrounded its progress. Researches vary in this respect; but of all those in which I have engaged, none has carried such a consistent accompaniment of delight, and I hope that some of this may be carried over in its presentation.

The possibility of determining the quantity of energy necessary for vision developed when Langley[1] invented the bolometer for measuring the energy distribution in the solar spectrum. Indeed, soon after this, Langley[2] himself tried to determine this minimum energy, and found it to be 3×10^{-9} erg for light of 550 mμ.

One must not lay too great stress on the specific value he obtained, nor on its accuracy. In view of the information which has since accumulated about the properties of the eye, we know that Langley's measurements were inadequate. Their significance, however, lies in the establishment of an approximate order of magnitude for this quantity of light.

The precise value of the minimum energy has become particularly significant since the advent of the quantum theory. The energy of a single quantum is 3.84×10^{-12} erg for light of 510 mμ. Therefore, even from Langley's measurements it follows that the number of quanta necessary for a minimal visual stimulus is not large. Since the sensitivity of the eye is dependent on the photodecomposition of visual purple, it would seem in terms of the Einstein equivalence law that a small number of primary molecular transformations constitutes the impetus for a biological act, in particular a biological act which involves the cortex of the brain and the other higher nerve centers.

Since Langley's day, about a dozen efforts have been made to redetermine this minimum energy.[3] It is to be regretted that many of these attempts contain serious errors which invalidate them. Indeed, in most of the researches no direct energy measurements were actually made; instead the investigators relied on previously measured energy distributions in standard sources, and made elaborate computations from them. This is understandable. Hardy and Perrin[4] say that "however simple in principle the measurement of radiation may appear to be, the amount of energy available is ordinarily so minute that the technique can be acquired only by long experience. In fact, there is probably no other type of measurement that contains so many pitfalls." Our own previous experience is a sad confirmation of this understatement.

Examination of all the published measure-

* This paper contains the substance of an address given at the Annual Dinner of the Optical Society of America and the Society of Rheology on October 24, 1941 at the Hotel Pennsylvania, New York City. Publication assisted by the Ernest Kempton Adams Fund for Physical Research of Columbia University.

[1] S. P. Langley, "The bolometer and radiant energy," Proc. Am. Acad. **16**, 342 (1881).

[2] S. P. Langley, "Energy and vision," Phil. Mag. [5] **27**, 1 (1889).

[3] In addition to Langley, these are: (a) G. Grijns and A. K. Noyons, "Ueber die absolute Empfindlichkeit des Auges für Licht," Arch. Anat. und Physiol. **25** (1905); (b) H. Zwaardemaker, "Die physiologisch wahrehmbaren Energiewanderungen," Ergeb. Physiol. **4**, 423 (1905); (c) J. von Kries and J. A. E. Eyster, "Über die zur Erregung des Sehorgans erforderlichen Energiemenzen," Zeits. Sinnesphysiol. **41**, 394 (1907); (d) H. E. Ives, "The minimum radiation visually perceptible," Astrophys. J. **44**, 124 (1916); (e) H. N. Russell, "The minimum radiation visually perceptible," Astrophys. J. **45**, 60 (1917); (f) P. Reeves, "The minimum radiation visually perceptible," Astrophys. J. **46**, 167 (1917); (g) H. Buisson, "The minimum radiation visually perceptible," Astrophys. J. **46**, 296 (1917); (h) P. Lecomte du Noüy, "Energy and vision," J. Gen. Physiol. **3**, 743 (1921); (i) J. Chariton and C. A. Lea, "Some experiments concerning the counting of scintillations produced by alpha particles. Part I," Proc. Roy. Soc. **A122**, 304 (1929); (j) H. A. Wentworth, "A quantitative study of achromatic and chromatic sensitivity from center to periphery of the visual field," Psych. Monograph **50**, 189 pp. (1930); (k) R. B. Barnes and M. Czerny, "Lässt sich ein Schroteffekt der Photonen mit dem Auge beobachten?" Zeits. f. Physik **79**, 436 (1932).

[4] A. C. Hardy and F. H. Perrin, *The Principles of Optics* (McGraw-Hill, New York, 1932).

ments on the energy required for vision shows that only three are free from what can now be recognized as obvious error. These are the data of Chariton and Lea which yield 0.66×10^{-10} erg, of von Kries and Eyster which give 1.3 to 2.6×10^{-10} erg, and of Barnes and Czerny which result in 1.5 to 3.3×10^{-10} erg. Even of these three, only the measurements of Barnes and Czerny involve actual energy determinations by the authors.

Neither von Kries and Eyster nor Chariton and Lea made any energy measurements. This does not mean that they made no measurements; they measured intensities and brightnesses and times and areas, but not energies. The conversion of these factors into final energies requires skill and care in the evaluation of absorptions, reflections, lens factors, and the like, and it is astonishing to see the admirable way in which von Kries, for example, accomplished this task.

The three most reliable sets of data differ by a factor of about 3. Nevertheless, they are of about the same order of magnitude, and in this respect must be considered as confirming one another. However, a factor of 3 cannot be ignored. Therefore, we undertook to make the measurements again. We chose what are now known to be the best physical and physiological conditions. We hoped that by knowing the values in a more definitive way, we could use them to yield information about the nature of the visual process.

II. PHYSIOLOGICAL CONDITIONS

The eye is not a uniformly sensitive surface and it is not equally sensitive under different conditions. There are circumstances which are bound to yield maximal retinal sensibility, and these conditions have been known for years. Indeed, one of the things which makes von Kries' and Eyster's values so reliable is their recognition of the precise physiological conditions necessary to achieve minimal values of this energy.

It is obvious that the eye must be thoroughly dark adapted. This means at least a 30-minute stay in the dark before measurements can be begun.[5] After complete dark adaptation the

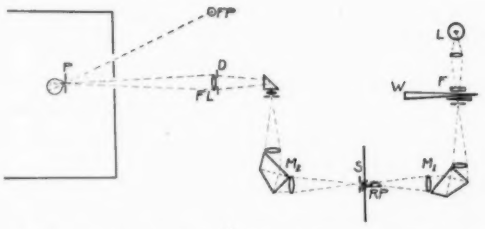

FIG. 1. Optical system for measuring minimum energies necessary for vision. The eye at the pupil P fixates the red point FP and observes the test field formed by the lens FL and the diaphragm D. The light for this field comes from the lamp L through the neutral filter F and wedge W, through the double monochromator M_1M_2 and is controlled by the shutter S.

periphery of the retina is much more sensitive than its center. The greatest density of rod elements begins at about 18° out,[6] and exploration shows that between 20° and 30° from the center there is a region of maximum sensibility to light.[7] The variation within this region is not large, and for convenience we chose an area situated 20° temporally on the horizontal axis.

It is known that the larger the area observed, the smaller need the intensity be for its recognition. This reciprocal relation is exact only for small areas. Our preliminary experiments, as well as the work of other investigators,[8] show a minimum for the product of area and intensity for fields of the order of 10 minutes diameter, and we therefore chose a circular retinal area of 10 minutes diameter for the test field.

There is an approximately reciprocal relationship between intensity and time of exposure for a visual threshold. This holds perfectly for values below 0.01 second.[9] To be sure of falling within this most efficient range, our exposures were 0.001 second long. Finally, from the measurements of the scotopic luminosity curve,[10] it is

[5] H. Piper, "Über Dunkeladaptation," Zeits. Psychol. u. Physiol. Sinnesorgane **31**, 161 (1903); S. Hecht, C. Haig, and A. M. Chase, "The influence of light adaptation on subsequent dark adaptation of the eye," J. Gen. Physiol., **20**, 831 (1937).

[6] G. Østerberg, "Topography of the layer of rods and cones in the human retina," Acta Ophth., Copenhagen, Supplement 6, pp. 106 (1935).

[7] H. A. Wentworth, see footnote 3 (j).

[8] For a summary of the field see G. Wald, "Area and visual threshold," J. Gen. Physiol. **21**, 269 (1938).

[9] C. H. Graham and R. Margaria, "Area and the intensity-time relation in the peripheral retina," Am. J. Physiol. **113**, 299 (1935).

[10] S. Hecht and R. E. Williams, "The visibility of monochromatic radiation and the absorption spectrum of visual purple," J. Gen. Physiol. **5**, 1 (1922).

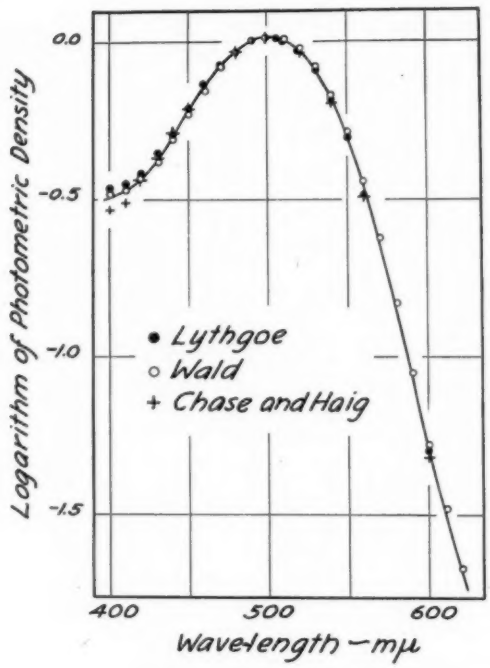

FIG. 2. Absorption spectrum of frog's visual purple. The data from the three sources have been made equal at 500 mμ.

known that for dim vision the eye is most sensitive to a wave-length of 510 mμ, and this is the light which we used for making the measurements.

III. PHYSICAL CONDITIONS

The physical arrangements may be seen in Fig. 1. The light source L is a ribbon filament lamp run on constant current from storage cells, and measured potentiometrically. By means of a lens, it is focused on the slit of a double monochromator M_1M_2 and finally on the artificial pupil P next to the eye of the subject who sits in a dark cabinet in a dark room. What the subject sees when he looks at the red fixation point FP is the uniformly illuminated field lens FL whose size is controlled by the diaphragm D.

The light intensity is varied in large steps by neutral glass filters F, and gradually by a wedge and balancer W, also of neutral glass. The filters and the wedge and balancer are calibrated in a spectrophotometer.

S is a precision shutter made of two parts. One part is merely a circular disk with a small sector of 10.8° removed and properly balanced. It is run at 1800 revolutions per minute by means of a synchronous motor. The other part is a polar relay shutter which covers the entrance slit of the second monochromator. By means of a phasing commutator on the shaft of the synchronous motor, this magnetic shutter is opened for only one passage of the rotating disk aperture whenever a push button switch is depressed. The control is in the hands of the observer so that he may at any time initiate a flash.

The energy at the pupil P was measured with a Hilger linear thermopile and a Paschen galvanometer. The thermopile and galvanometer had been calibrated with a standard carbon filament lamp under the same conditions as those prevailing in the apparatus. The measurements were made on two occasions several months apart with excellent agreement.

IV. MEASUREMENTS

From the observer's point of view, an experiment involves the report of whether or not he has seen a flash of light after he has opened the shutter for an exposure. Fixation of the red point is not continuous, a circumstance which avoids undue fatigue. The observer is told by the operator that conditions are set and that he should try a flash when he is ready. He fixates the red point, and at the moment which he

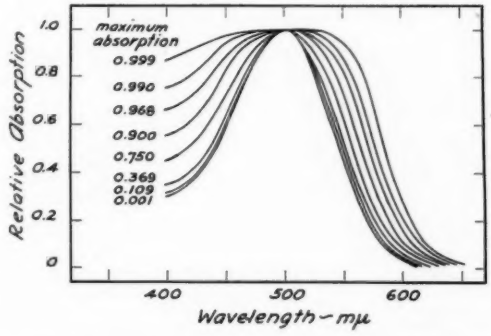

FIG. 3. Percentage absorption spectra of various concentrations of visual purple. For convenience in comparing the shapes of the curves, their maxima have all been equated to 1 and are thus superimposed at 500 mμ. The actual fraction absorbed at the maximum is shown for each curve. It is apparent that with increasing concentration the absorption curve steadily increases in width.

considers propitious, he exposes the light to his eye. The operator changes the position of the wedge, or removes or introduces a filter until he is satisfied with the precision of the measurements. In general, a threshold is considered reached when the observer records that he sees a flash of light six times out of ten presentations.

We have made measurements with six observers in the laboratory during a period of several months, and find that the minimum energy necessary for vision ranges between 2.2 and 5.7×10^{-10} erg at the cornea. These small energies represent between 58 and 148 quanta of blue-green light.

It is to be noticed that these energies are of the same order of magnitude as those of von Kries and Eyster, and of Barnes and Czerny, but almost twice as large. Because of the fairly wide ranges, these previous measurements and our own overlap to some extent, and it is conceivable, though not probable, that their observers may actually have needed somewhat smaller energies than ours. Chariton and Lea's values, however, are much too small, and are apparently in error.

V. CORRECTIONS

These values are for energies incident at the cornea, and the tacit supposition has generally been made that they represent the energy necessary for a visual act. It is important to recognize that this assumption is incorrect. Before one can know how many quanta are required to initiate a visual act, one must apply at least three corrections to the measurements.

The first is reflection from the cornea. This is about 4 percent and is obviously of not much importance. The second involves loss by the ocular media between the outer surface of the cornea and the retina. It has been common opinion that this loss is small. However, the measurements of Roggenbau and Wetthauer[11] on cattle eyes, as well as the recent measurements of Ludvigh and McCarthy[12] on human eyes, have shown that this loss is large. From the values of Ludvigh and McCarthy it appears that at 510 mμ the ocular media transmit almost exactly 50 percent of the light entering the cornea of a young person, and less of an older one.

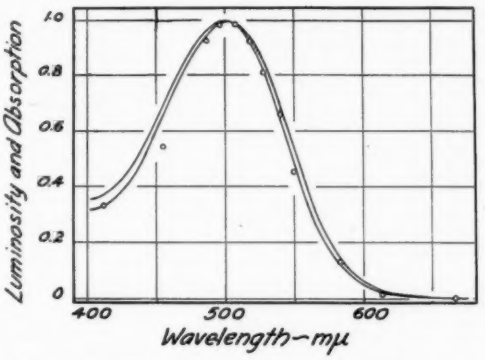

FIG. 4. Comparison of scotopic luminosity at the retina with visual purple absorption. The points are the data of Hecht and Williams corrected for quantum effectiveness and ocular media transmission. The curves are the percentage absorption spectra of visual purple; the upper curve represents 20 percent maximal absorption, and the lower one 5 percent maximal absorption. All curves have been made equal to 1 at the maximum, 500 mμ, for ease in comparison.

The next correction is perhaps more difficult to evaluate with precision and involves the percentage of the energy absorbed by the retinal elements themselves. Since visual purple is the photosensitive substance for this particular job, light which is not absorbed by it is visually useless. One cannot assume that visual purple absorbs all the light incident on the retinal cells. The fraction which it does absorb must be found by experiment.

Koenig[13] determined the absorption of the total amount of visual purple which can be extracted from the human eye. If this amount of visual purple is spread evenly over the whole retina, his data show that it will absorb only 4 percent of light of 510 mμ. This is a small value. Nevertheless, it is about the same as the 4 percent and the 13 percent recently found by Wald[14] with a similar method for the absorption of the visual purple of the rabbit and rat retinas, respectively.

[11] C. Roggenbau and A. Wetthauer, "Über die Durchlässigkeit der brechenden Augenmedien für langwelliges Licht nach Untersuchungen am Rindsauge," Klin. Monatsbl. Augenheilk. **79**, 456 (1927).

[12] E. Ludvigh and E. F. McCarthy, "Absorption of visible light by the refractive media of the human eye," Arch. Ophthal. **20**, 37 (1938).

[13] A. Koenig, "Ueber den menschlichen Sehpurpur und seine Bedeuting für das Sehen," Sitzungsb. Akad. Wissensch., Berlin, 577 (1894).

[14] G. Wald, "On rhodopsin in solution," J. Gen. Physiol. **21**, 795 (1938).

These figures are probably too low, first because it is unlikely that all of the visual purple in the eye has been extracted, and second, because visual purple is not evenly distributed over the retina. It is lacking in the fovea; and even in the periphery the density of the rods is known to vary in a definite way. However, these absorptions may be considered as lower limiting values.

VI. VISUAL PURPLE ABSORPTION

We have estimated the absorption of visual purple in the retina in a completely independent manner by comparing the percentage absorption spectrum of different concentrations of visual purple with the scotopic luminosity curve of the eye measured at the retina. The comparison rests on the fact that the shape and width of the percentage absorption spectrum of a substance varies with its concentration, and that the luminosity curve must represent the percentage absorption curve of a particular concentration of visual purple in the retina.

Figure 2 shows the absorption spectrum of frog's visual purple as determined by Chase and Haig[15] in our laboratory, by Lythgoe[16] in London, and by Wald[14] at Harvard. The agreement of the data is obvious, and shows that the absorption spectrum of visual purple may be considered as well established. From these data we may prepare a series of percentage absorption spectra for different concentrations of visual purple. Since we are not interested in the absolute concentration of visual purple, but rather in its absorption capacities, we can deal with the series of percentage absorption spectra entirely in terms of maximum absorption. Figure 3 shows the behavior of the percentage absorption curve for different maximal absorptions of visual purple. For comparison, the maxima have all been made equal to 1, but their values are indicated.

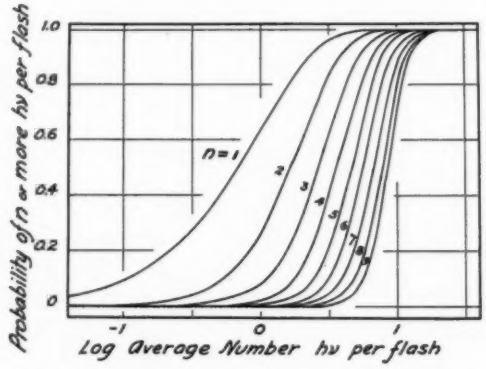

FIG. 5. Poisson probability distributions. For any average number of quanta per flash, the ordinates give the probabilities that the flash will deliver to the retina n or more quanta, depending on the value assumed for n.

The scotopic luminosity curve, as measured experimentally, records the reciprocal of the relative energy in different parts of the spectrum required for the production of a constant and very low brightness in the eye.[10] If this is to be compared with the absorption spectrum of visual purple, it must be converted into a quantum luminosity curve instead of an energy luminosity curve, because it is the number of quanta which determines the photochemical effectiveness of light and not just its energy content.[17] Moreover, since our interest lies in retinal comparisons, the luminosity curve must be corrected for ocular media absorption in terms of the data of Ludvigh and McCarthy.

The scotopic luminosity data have been corrected in these two ways, and the computed values are shown as circles in Fig. 4. Included in the same figure are two percentage absorption spectra of visual purple; the upper curve represents 20 percent maximal absorption, while the lower curve is 5 percent maximal absorption.

For comparing the luminosity and absorption data, it is well to confine our attention mostly to the long wave half of the luminosity curve because of the larger number of points involved. From the comparison it is apparent that the 5 percent maximum absorption curve describes the points quite well, but that the 20 percent curve is definitely excluded, because its absorption on both sides is just too high. The 10 percent absorption curve, not shown in the figure, is perhaps slightly better than the 5 percent one; it cuts through more points. In any case, both

[15] A. M. Chase and C. Haig, "The absorption spectrum of visual purple," J. Gen. Physiol. **21**, 411 (1938).

[16] R. J. Lythgoe, "The absorption spectra of visual purple and of indicator yellow," J. Physiol. **89**, 331 (1937).

[17] H. J. A. Dartnall and C. F. Goodeve, "Scotopic luminosity curve and the absorption spectrum of visual purple," Nature **139**, 409 (1937).

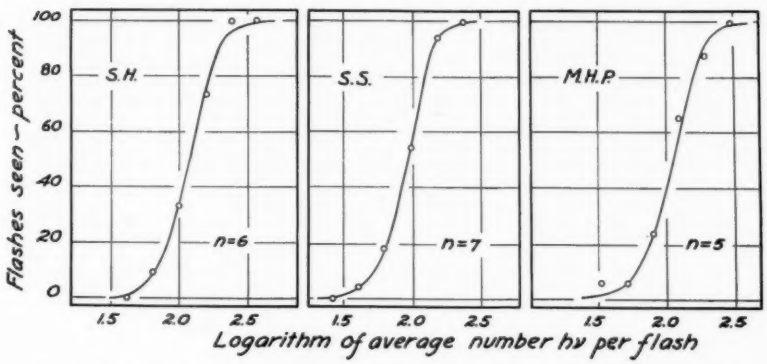

FIG. 6. Relation between the average energy content of a flash of light and the frequency with which it is seen by three observers. Each point represents 50 flashes. The curves are the Poisson distributions of Fig. 5 for n values of 5, 6, and 7.

values are of the same order of magnitude as those found by Koenig and by Wald. However, to be quite safe, we may take 20 percent as the upper limit for the absorption of λ510 mμ by visual purple.

VII. ENERGY ABSORBED

If we now apply these three corrections to the minimum perceptible energy, the range of 58 to 148 quanta at the cornea becomes as an upper limit 5 to 14 quanta absorbed by the visual purple of the retina.

Visual purple is contained in the terminal segments of the rods, and our 10-minute circular visual field contains about 500 rods. It is therefore unlikely that any one rod will take up more than one quantum out of the small number absorbed. In fact, the probability that 2 quanta will be taken up by a single rod cell is only about 4 percent. We may therefore conclude that in order to see, it is necessary for only 1 quantum of light to be absorbed by each of 5 to 14 retinal rods.

It is very likely that the photodecomposition of visual purple in solution[18] has a quantum efficiency of 1. Our data then mean that 1 molecule of visual purple needs to be changed simultaneously in each of 5 to 14 rods, in order to produce a visual effect. This is indeed a small number of chemical events, but by virtue of its very smallness, its reality may be tested in an entirely independent manner.

VIII. POISSON DISTRIBUTIONS

The energy calibration of the light gives merely the average number of quanta per flash. This is in the nature of the measurement, because the thermopile records the number of quanta per second from a continuously incident light. Each flash, however, will not always deliver this average number. Sometimes the flash will yield fewer, sometimes more, quanta.

Since absorption of this group of quanta by the retina represents discrete and independent events which occur individually and collectively at random, the actual number of such retinal events which any given flash provides will vary according to a Poisson probability distribution.[19] Let n be the number of quanta which it is necessary for the retina to absorb in order for us to see a flash of light. Let a be the average number of quanta which any flash yields to the retina. Then according to the Poisson distribution, the probability P_n of the flash yielding the necessary n quanta is given by the equation

$$P_n = a^n e^{-a}/n!$$

A special virtue of the Poisson distribution is that it has only one parameter, and thus its value is at once given when the average number a is set.

[18] H. J. A. Dartnall, C. F. Goodeve, and R. J. Lythgoe, "The effect of temperature on the photochemical bleaching of visual purple solutions," Proc. Roy. Soc. **A164**, 216 (1938).

[19] T. C. Fry, *Probability and its Engineering Uses*, (Van Nostrand, New York, 1928).

Since for us to see a flash of light the retina must absorb n quanta, we shall also see if the retina absorbs more than n quanta. From the Poisson distributions one can then compute the probability that n or more quanta will be delivered to the retina in a given flash when the average number of quanta delivered by that flash is known. The values computed in this way for different values of a and n are shown in Fig. 5.

There are two significant features of Fig. 5. One is that the shape of the distribution is fixed and different for every value of n. The curve becomes steeper as n increases. It follows from this that if the probability distribution could be determined by experiment, its shape would automatically reveal the value of n corresponding to it.

Another and equally important feature of Fig. 5 is that the relationship is expressed in terms of the logarithm of the average number of quanta per flash. Therefore, for comparison with the distributions in Fig. 5, the experiments need not employ the absolute values of the average number of quanta delivered per flash, but merely their relative values.

The experiments may then be made quite simply. On many repetitions of a flash of given average energy content, the frequency with which the flash is seen will depend on the probability with which it yields n or more quanta to the retina. When this frequency is measured for each of several intensities, a distribution is secured whose shape, when plotted against the logarithm of the average energy content, should correspond to one of the probability distributions in Fig. 5, and should thus show what the value of n has been.

IX. FREQUENCY MEASUREMENTS

We have made determinations of this kind. The experimenter varies the intensity of the light by placing the wedge in specific positions unknown to the observer. The observer then elicits the flash whenever he is ready, and merely reports whether he has seen it or not. The intensities are presented in a deliberately random sequence, each for 50 times. A series in which six intensities are used requires about one and one-half hours of continuous experimentation in two or perhaps three periods of intensive work.

The comfort of the observer is of great importance and this must be at a maximum. It is equally important that fixation should not be continuous because this is fatiguing, and the observer must be on guard to record any subjective feelings of fatigue the moment they become apparent. The experiment is much facilitated by the fact that the observer controls the occurrence of the flash, and can set it off only when he is thoroughly fixated and ready for an observation.

The data for the three observers who engaged in this experiment are given in Fig. 6. They record the percentage frequency with which a flash of light is seen for flashes of average quantum content shown in the abscissas. Comparison with the curves in Fig. 5 shows that the measurements are best fitted by Poisson distributions in which n is 5, 6, and 7 quanta per flash. No special statistical methods are necessary to determine which curve fits the data, since smaller and larger values of n are easily excluded by the simplest visual comparison.

From these measurements it is apparent that the number of critical events in the retina required to produce a visual effect lies between 5 and 7. These values are in such good agreement with the results determined by the straightforward physical measurements already described that we must consider them as the actual number of quanta absorbed by the retina.

It is unimportant that the intensities delivered to the cornea are very much higher than these values since we have seen that most of the light incident on the cornea does not ultimately contribute to the initiation of a visual act. The amount falling on the cornea could be greatly increased by any arrangement in the eye which would act as a filter. Thus, the cornea and the lens might be pigmented. Indeed, one might even put a filter in front of the eye. The probability distributions would still remain the same, and by their shape would yield the magnitude of the number of events involved in the visual act.

X. BIOLOGICAL VARIATION

The one assumption which we have made in applying the Poisson probability distribution is that a constant number of quanta n is necessary to be absorbed by the retina in order for us to

see a flash of light. It is conceivable, in view of the variability of an organism from moment to moment, that this value n is not constant. We have considered the consequences of assuming that the number n varies from time to time, and the results show that biological variation is a factor of no great importance.

The situation may be best made clear by an example. Suppose that instead of n being constant, it varies between 5 and 9 quanta per visual act, and that the frequency with which 5, 6, 7, 8 and 9 quanta are necessary is distributed in terms of an ordinary probability distribution. The curves in Fig. 5 representing the frequency distributions for various values of n may then be weighted in this way and averaged. The average curve which is then secured is practically the same as the original Poisson distributions in Fig. 5, and may be fitted by the curves for $n=5$ or 6.

Thus, when biological variation is imposed upon the physical variation, there is no change in the essential characteristics of the physical distribution. Instead, the value of n merely falls below the average of the biological distribution, and is never below the lowest value in the distribution. This tells us that when, as in Fig. 6, the measurements yield n values of 5, 6, or 7, these numbers represent lower limiting values for the physical number of quanta. In other words, the only effect which biological variation has on the physical variation is to decrease the slope of the curves in Fig. 6 and thus make the apparent number of quanta smaller than the real number.

These considerations are significant for an understanding of the nature of the fluctuations shown by an organism in its response to a stimulus. It has generally been assumed that a constant stimulus, when presented frequently, remains constant, and that the fluctuations in response are an expression of the variations undergone by the organism. Indeed, this is one of the tenets of psychological measurement, and an elaborate structure of psychometrics has grown up on it as a basis.[20]

The present evaluation of our measurements shows, however, that at the threshold the emphasis has been in the wrong place. At the threshold where only a few quanta of energy are involved, it is the stimulus which is variable, and the very nature of this physical variability determines the variation encountered between response and stimulus. Moreover, even when biological variation is introduced, it is the physical variation which essentially dominates the relationship.

It is well to emphasize that this is at the absolute threshold. One may speculate, however, as to whether a differential threshold at any level of intensity may also involve a small number of events which determine the differentiation, and which may therefore be subject to a similar physical variation as at the absolute threshold itself. I record this speculation merely to indicate that experiments are at present under way to explore its possibilities.

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[20] J. P. Guilford, *Psychometric Methods* (McGraw-Hill, New York, 1936).

By-Laws of the Optical Society of America, Inc.

ARTICLE I

MEMBERSHIP

Section 1. Classes of Membership: Membership in the society shall be regular, associate, student, corporation, emeritus, or honorary.

Section 2. Charter Members: The members of the unincorporated association, the Optical Society of America, at the time of the incorporation of said society shall be members in their respective classifications of the Optical Society of America, Inc. The charter members of the unincorporated society shall be known as the charter members of the incorporated society.

Section 3. Eligibility: Any person who has contributed materially to the advancement of optics is eligible for regular membership.

Any person who is interested in optics is eligible for associate membership.

Any *bona fide* student, undergraduate or graduate, of an educational institution of collegiate grade who is devoting more than half-time to activities within the institution is eligible for student membership.

Any corporation desirous of encouraging the work of the society is eligible for corporation membership.

Any person who has retired from active work, has reached the age of 65 years, and has been a member for at least 20 years is eligible for emeritus membership. Other members may, by a two-thirds (2/3) vote of the Board of Directors, be declared eligible for emeritus membership.

Any person who has rendered eminent service in the advancement of optics may be elected to honorary membership.

Section 4. Application for Membership: Applications for student membership must include a statement of the student's academic status and must be signed by one associate, regular, emeritus, or honorary member of the society. The correctness of the statements in the application must be certified by an officer or faculty member of the educational institution. Other applications for membership must state the candidate's name and qualifications for membership and must be signed by two associate, regular, emeritus, or honorary members of the society.

Section 5. Election to Membership: Applicants for student membership shall become members when their duly prepared applications have received the written approval of two members of the board of directors. All other applications from individuals for membership shall be considered first as applications for associate membership and shall be acted upon as such. Applicants shall become associate members when their duly prepared applications have received the written approval of three members of the board of directors.

Applicants for corporation membership shall become corporation members when their duly prepared applications have received the written approval of three members of the board of directors.

Student members may be transferred to associate membership by vote of the board of directors.

Associate members may be transferred to regular membership at the regular annual meeting of the board of directors by a two-thirds (2/3) vote of the members of the board present.

Any eligible member may, at his own request, be transferred to emeritus membership by the Secretary.

Honorary members may be elected at any regularly called meeting of the board of directors by the unanimous vote of the entire board. In case any members of the board are not in attendance their ballots must be obtained by the secretary. The number of honorary members shall at no time exceed five (5).

Section 6. Duties and Privileges: Regular members shall have the right to vote, hold office, attend meetings, and receive the journal of the society and such other publications as the board of directors may direct.

Student and associate members shall have the same privileges as regular members except that they may not vote or hold office.

Corporation members shall receive the journal of the society and such other publications as the board of directors may direct. Their representatives may attend all meetings of the society.

Emeritus members shall have all of the privileges of their former membership status, except

the right to receive the Journal and other publications of the Society.

Honorary members shall have all of the privileges of regular members.

Section 7. Termination of Membership: Membership in the society may be suspended or terminated by vote of the board of directors. Failure to pay dues for one (1) year shall be deemed sufficient cause for suspension or termination of membership.

Student membership in the society shall terminate at the end of the calendar year unless the student requests continuation of his student membership. Such a request must include a statement of the student's eligibility, and the correctness of the statement must be certified by an officer or faculty member of the educational institution.

Section 8. Dues: The dues for regular members shall be seven dollars and fifty cents ($7.50) per annum.

The dues for associate members shall be five dollars ($5.00) per annum.

The dues for student members shall be two dollars ($2.00) per annum.

The dues for corporation members shall be fifty dollars ($50.00) per annum.

The payment of one hundred dollars ($100.00) in one sum shall exempt any regular or associate member from the subsequent payment of dues for the remainder of his life.

Section 9. Patrons: The payment, within any one year, of one thousand dollars ($1000.00) or more toward the expenses of the society by any corporation or individual shall entitle such corporation or individual to the rank of patron of the society, for a period of ten years in the case of a corporation, and for life in the case of an individual.

Section 10. Ownership of Corporation Property: The corporation shall not be operated for profit and its entire properties, assets, and facilities shall be devoted to the purposes for which it is organized as set forth in its certificate of incorporation as the same may be from time to time amended. No member shall have any right to or any share or interest in any of the property or assets of the corporation.

Section 11. Student Branches: A student branch of the Optical Society of America may be formed at an educational institution of collegiate grade on application by a faculty member or officer of the institution who is also an associate, regular, or honorary member of the society. Such application must be approved by the board of directors.

A student branch shall receive one copy of each notice and program of the society. The society shall not assume any financial obligation to a student branch nor financial responsibility for the actions of that branch.

Article II

Meetings of Members

Section 1. Annual Meeting of the Corporation: During October of each year at a time and place to be fixed by the board of directors, there shall be held the annual meeting of the members of the corporation for the election of directors and officers (if not previously elected by mailed ballot as hereinafter provided), and for the rendering to the membership of the annual report of the directors as required by law, and for the transaction of such other business as may properly come before the meeting.

At each annual meeting the directors shall present a report verified by the President and Treasurer or by a majority of the directors, showing the whole amount of real and personal property owned by the corporation, where located and where and how invested, the amount and nature of the property acquired during the year immediately preceding the date of the report, and the manner of the acquisition; the amount applied, appropriated or expended during the year immediately preceding such date, and the purposes, objects, or persons to or for which such applications, appropriations or expenditures have been made; and the names and places of residence of the persons who have been admitted to membership in the corporation during such year, which report shall be filed with the records of the corporation and an abstract thereof entered in the minutes of the annual meeting.

Section 2. Notice of Annual Meeting: Notice of the annual meeting of members shall be given by mailing, not less than ten (10) days nor more than forty (40) days before the meeting, a copy of the notice of such meeting to every member of the

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corporation directed to him at his address as it appears upon the records of the corporation.

Section 3. Special Meetings: Special meetings of the members, except as otherwise provided by law or by these by-laws, may be called at any time by the board of directors or by the President, and shall be called by the President or by the Secretary at the request in writing of a majority of the board of directors. Such request shall state the purpose or purposes of the proposed meeting.

Section 4. Notice of Special Meetings: Notice of each special meeting of the members shall be given by mailing, not less than ten (10) nor more than forty (40) days before the meeting, a copy of the notice of such meeting, stating the purpose or purposes for which the same is called, to each member directed to him at his address as it appears on the records of the corporation.

Section 5. Quorum: At any meeting of the members of the corporation a quorum shall consist of not less than twenty (20) voting members.

Section 6. Qualifications of Voters: Upon any question presented for vote to the membership, whether presented by mail or at a meeting of the members, each member entitled to vote pursuant to the provision of Section 6 of Article I shall be entitled to one (1) vote.

Section 7. Voting: Upon any question with respect to which a vote of the membership shall be required or deemed advisable, the board of directors may determine whether such vote shall be taken by mail or by ballot at the next meeting of the membership. In case the directors shall determine to take such vote by mail, either upon any question of administration or policy or upon the election of officers, directors or others, forms for the casting of such vote or for the casting of a ballot for the election of any such officers, directors, or others, shall be forwarded to the entire membership entitled to vote thereon at least thirty (30) days prior to the date upon which the count of such votes or ballots shall be made by the directors or tellers appointed for the purpose (the date of which count shall be incorporated in the advice sent to the respective members) and the results of such vote or ballot shall be determined by the directors or tellers upon the date fixed for such purpose and shall be valid, binding and of full force and effect precisely as if such vote had been taken in open meeting of the membership regularly called and duly held, provided always, that in order to render valid such vote by mail, the number of votes cast shall be at least equal to the number of voting members required to constitute a quorum at any regular or special meeting of the membership.

Article III

Officers and Directors

Section 1. Officers Enumerated: The officers of the society shall be a President, a Vice President, a Secretary, a Treasurer, a Junior Past President, and an Editor of Publications.

Section 2. Directors Enumerated: The directors shall consist of the six (6) officers mentioned in Section 1, all of whom shall be, ex-officio, members of the board of directors, and five (5) directors-at-large.

Section 3. Eligibility and Election: Nine (9) directors and officers shall be elected by the voting members from among the regular, honorary, and emeritus members of the society. These directors shall consist of five (5) directors-at-large and the following officers of the society who shall be, ex-officio, members of the board of directors, to wit: president, vice president, secretary, and treasurer.

At the expiration of his term of office the president becomes the junior past president, one of the officers of the society, and ex-officio, a member of the board of directors.

The editor of publications shall be elected by the above-enumerated board of directors at an annual meeting of the board and shall upon such election become a voting member of such board.

Section 4. Terms of Office: The president and vice president shall be elected for terms of two (2) years.

The secretary and the treasurer shall be elected for terms of four (4) years.

No officer elected by the membership shall be eligible for re-election except when such eligibility is established by special action of the board of directors. This shall require an affirmative vote of a majority of the entire board.

The junior past president shall serve until such time as his successor qualifies for the office by retiring from the office of president.

The editor of publications shall be elected for a term of four (4) years.

Members-at-large of the board of directors shall be elected for terms of four (4) years, except that at the first election following the formation of this corporation two (2) of these directors-at-large shall be elected for terms of two (2) years. At the expiration of the terms of these two directors-at-large their successors shall then be elected for terms of four (4) years as provided in the first sentence of this paragraph.

No member-at-large of the board of directors shall be eligible for re-election.

The term "re-election" as used in these by-laws shall be construed to mean only the election of an individual to succeed himself.

Section 5. Time of Election and Assumption of Duties: Election by the members of officers and members-at-large of the board of directors shall take place at an annual meeting of members in October, or by mailed ballot as provided in the following section.

The officers and directors elected shall assume their duties at the close of such annual meeting.

Section 6. Mode of Election: Elections shall be by ballot cast by mail as provided in Section 7 of Article II unless the board of directors shall, prior to the holding of any annual meeting of the membership, determine to hold such election at such annual meeting, in which case the members entitled to vote shall be notified of such determination. Nominations shall be made by a nominating committee appointed by the President and approved by the board of directors. This committee shall consist of five (5) regular members of the corporation, at least one (1) of whom shall be a past president.

Notices of the nominations made by the nominating committee shall be sent to all regular members at least thirty (30) days before the date on which the ballots are forwarded to the members. Additional nominations may be made by request of ten (10) members qualified to vote provided they are forwarded to the Secretary within twenty (20) days after notices of the nominations are sent out.

Section 7. Vacancies: In the event of a vacancy occurring among the officers or the directors-at-large, the remaining directors, by an affirmative vote of a majority thereof, may fill such vacancies for the unexpired term of office.

Section 8. Organization of the Board of Directors: The President of the society shall be the chairman of the board of directors.

The Secretary of the society shall be the secretary of the board of directors. He shall keep complete and precise records of the business transacted by the board of directors and shall send copies of the minutes to each member of the board and shall file copies of the minutes in a permanent record book provided for the purpose.

Section 9. Meetings of the Directors: The annual meeting of the board of directors shall be held in October. Meetings of the directors may be held at such other times as the chairman of the board may appoint. The directors may adopt rules and regulations governing their procedure, the notices to be given of meetings, the times and places of holding of regular meetings, and other matters with respect to the conduct of their business. The vote of directors upon any proposition, except as otherwise provided by these by-laws, may be obtained by mail during an interim between meetings.

Section 10. Quorum: At any meeting of the board of directors a majority of the entire membership of the board shall constitute a quorum and, except as otherwise provided by these by-laws, a majority of such a quorum shall decide any question that may come before the meeting.

Section 11. Duties of Officers: The duties of the President, Vice President, Secretary, and Treasurer of the society shall be the usual ones performed by such officers.

The Treasurer shall be chargeable with the responsibility for the general funds of the corporation and for such special funds as may be from time to time placed in his custody or control by order of the board of directors, and he shall pay the bills of the corporation which have been approved by the board of directors either through adoption of an annual budget or by special action.

The custody of other special funds may be vested by the board of directors in agents other than the Treasurer.

The Treasurer and each other officer or agent having custody or control of any funds shall report periodically and at least once each year to the board of directors with respect to the manage-

ment and disposition of the fund or funds with which he is chargeable, and the board of directors shall in turn incorporate such reports in their annual report to the members.

The Editor of Publications shall have general charge under the supervision of the board of directors, of the publications of the society as set forth in Section 1 of Article V.

Section 12. Bonding: The Treasurer and each other officer or agent having custody or control of any funds of the corporation shall be bonded for an amount to be fixed by the board of directors.

Section 13. Prohibitions on Officers: No director or other officer of the society shall receive, directly or indirectly, any salary, compensation or emolument from the corporation, either as such officer or director or in any other capacity, unless authorized by the by-laws of the society, or by the affirmative vote of two-thirds (2/3) of all the directors.

No director or other officer of the society shall be interested, directly or indirectly, in any contract relating to the operations conducted by the society, nor in any contract for furnishing supplies thereto, unless authorized by the by-laws of the society or by the affirmative vote of two-thirds (2/3) of the directors.

Article IV

Committees and Delegates

Section 1. Committees: The President may appoint, for a term not to exceed the remainder of his term of office and subject to the approval of the board of directors, such committees as may further the objects of the Society, and such committees shall have such duties and responsibilities as shall be delegated to them by the President and the board of directors, but shall be subject at all times to the direction and control of the board of directors. Unless otherwise specifically provided in its appointment, each such committee shall serve until it has been discharged or its successor has been appointed.

Section 2. Representatives and Delegates: The President shall submit to the board of directors nominations for representatives and delegates to other organizations. Representatives and delegates shall be elected by the board of directors and for such terms as their respective offices may require. Unless otherwise specifically provided in his appointment each such representative or delegate shall serve until he has been discharged or his successor has been appointed.

Article V

Publications and Endowments

Section 1. Publications: The society shall publish a journal devoted to the advancement of optics and may publish such other scientific publications as may be deemed advisable by the board of directors from time to time. In addition to the Editor of Publications, who shall be a member of the board of directors as hereinabove provided, the board of directors may appoint six or more associate editors whose terms of office shall be six (6) years, one-third (1/3) of them to be appointed biennially, but such appointments to be at all times subject to revocation by the board of directors. Every regular, associate, student, corporation, and honorary member of the society shall be entitled to receive the journal of the society.

Section 2. Endowments: The corporation may accept endowments for medals, scholarships, or for other purposes, and such endowments shall be maintained in accordance with the directions of the board of directors and the wishes of the donors.

Article VI

Finances

Section 1. Finances: The respective funds of the corporation shall be deposited with such bank or banks, trust company or trust companies as the board of directors may from time to time designate, and shall be subject to withdrawal by such officers or agents as may from time to time be designated by the board of directors.

Article VII

Corporate Seal

Section 1. Form of Seal: The seal of the corporation shall be circular in form with the words "Optical Society of America, Inc., New York" in the outer circle and "1932" together with the official emblem of the society within the inner

circle. Such seal as is impressed on the margin hereof is hereby adopted as the corporate seal of the corporation.

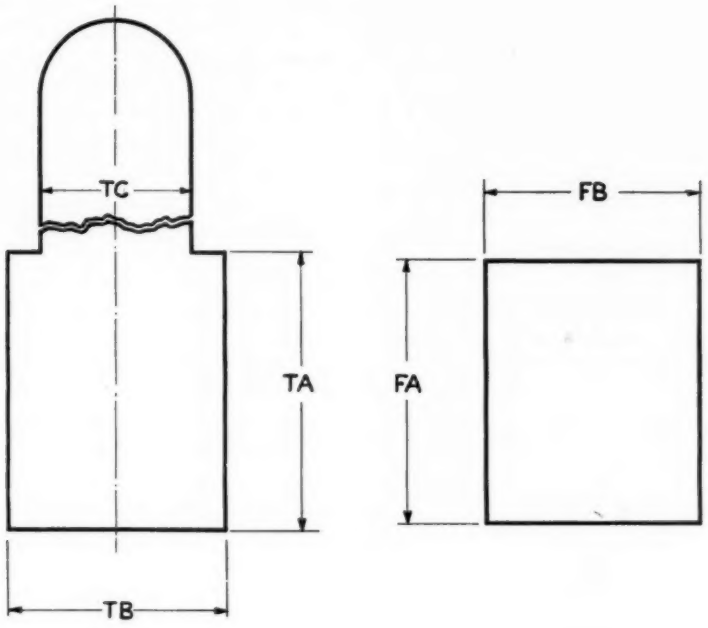
Article VIII

Amendments

Section 1. Amendments by Directors: These by-laws may be amended, altered, or repealed in whole or in part by the affirmative vote of the majority of the entire board of directors expressed in writing or cast at any regular or special meeting of the board with or without mention thereof in the notice of the meeting at which the vote is taken. If, however, any such amendment is adopted at a meeting without mention thereof in the notice of such meeting, a copy of such amendment shall be mailed immediately after the meeting to each director not present at the meeting.

Section 2. Amendments by Members: These by-laws may be amended, altered, or repealed in whole or in part by the affirmative vote of a majority of the regular members of the society in good standing expressed in writing as provided in Section 7 of Article II or cast at any annual meeting of the members or at any special meeting of the members where such proposed action has been incorporated in the notice of the meeting. Amendments by the members may be made notwithstanding any previous action of the board of directors.

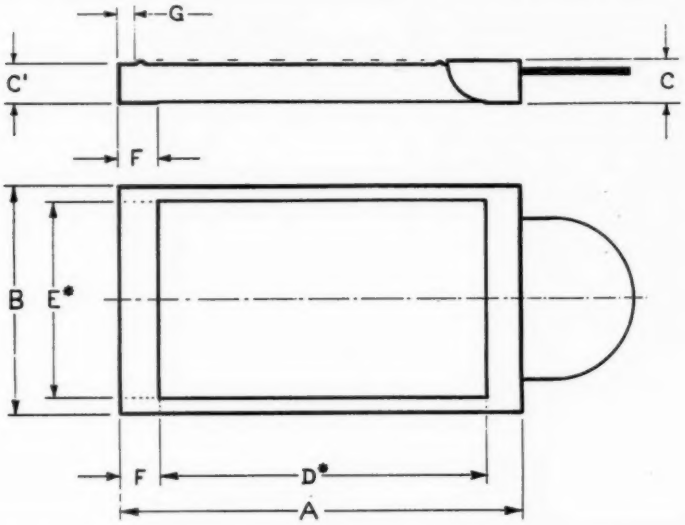
American Standard Dimensions Z38.1.1–1941 for Film Pack Tabs and Films*†



American Standard															
Nominal Film Pack Size	Tab						Film								
	TA		TB		TC		FA				FB				
	max.		max.		max.		min.		max.		min.		max.		
	mm	inch	mm	inch	mm	inch	mm	inch	mm	inch	mm	inch	mm	inch	
4.5×6 cm 1¾×2⅜ inch	76.6	3.016	46.5	1.831	35.0	1.378	71.0	2.795	74.0	2.913	44.5	1.752	46.0	1.811	
6×9 cm 2¼×3½ inch	96.0	3.780	65.5	2.579	50.0	1.968	90.0	3.543	93.0	3.661	63.5	2.500	65.0	2.559	
8×10.5 cm 3¼×4¼ inch	116.5	4.587	84.0	3.307	70.0	2.756	111.0	4.370	114.0	4.488	81.6	3.212	83.0	3.268	
8×14 cm 3¼×5½ inch	156.5	6.161	89.0	3.504	76.0	2.992	150.0	5.906	153.5	6.043	87.0	3.425	88.5	3.484	
9×12 cm 3½×4¾ inch	130.5	5.138	90.5	3.563	73.0	2.874	124.0	4.882	127.5	5.020	87.5	3.445	89.0	3.504	
10×12.5 cm 4×5 inch	138.5	5.453	103.5	4.075	88.0	3.465	132.5	5.217	136.0	5.354	100.5	3.957	102.0	4.016	
10×15 cm 4×6 inch	158.5	6.240	101.5	3.996	84.0	3.307	152.0	5.984	155.5	6.122	98.5	3.878	100.0	3.937	
13×18 cm 5×7 inch	188.5	7.421	129.0	5.079	110.0	4.331	182.0	7.165	186.5	7.343	126.5	4.980	128.0	5.039	

* This publication is one of a series of American Standards for Photography (other than Cinematography).
† Approved June 20, 1941, by the American Standards Association, 29 West 39 Street, New York, New York. The Optical Society of America is sponsor for the project.

American Standard Dimensions Z38.1.2–1941 for Film Pack Cases*†



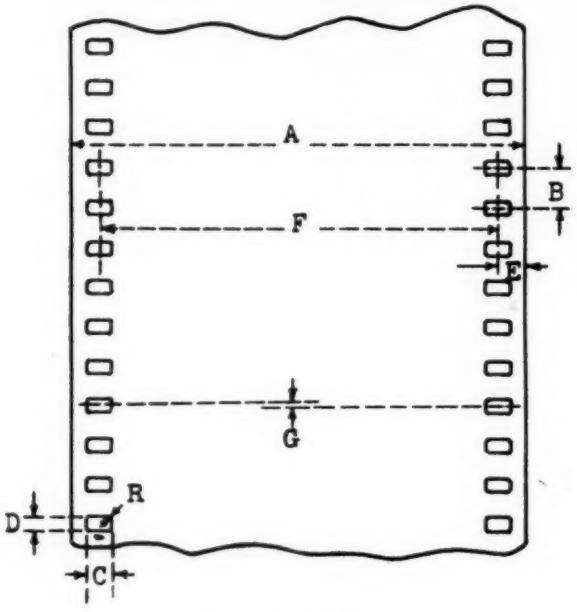
Dimensions in Millimeters and Inches		Film Pack Size							
		4.5×6 cm 1¾×2¾ inch	6×9 cm 2½×3½ inch	8×10.5 cm 3¼×4¼ inch	8×14 cm 3¼×5½ inch	9×12 cm 3½×4¾ inch	10×12.5 cm 4×5 inch	10×15 cm 4×6 inch	13×18 cm 5×7 inch
American Standard									
A max.	mm	85.0	107.0	130.0	168.0	142.7	150.5	170.5	200.5
	inch	3.346	4.213	5.118	6.614	5.618	5.925	6.713	7.894
B max.	mm	50.0	69.3	87.0	92.6	94.0	106.5	104.5	132.5
	inch	1.968	2.728	3.425	3.646	3.701	4.193	4.114	5.217
C max.	mm	13.2	13.2	13.2	13.2	13.2	13.2	13.2	13.2
	inch	.520	.520	.520	.520	.520	.520	.520	.520
C′ max.	mm			12.7	12.7		12.7		12.7
	inch			.500	.500		.500		.500
D min.	mm	60.0	80.5	101.0	137.5	113.5	122.5	140.5	171.5
	inch	2.362	3.169	3.976	5.413	4.468	4.823	5.531	6.752
E min.	mm	40.5	58.5	76.5	82.0	82.5	94.5	93.5	119.0
	inch	1.594	2.303	3.012	3.228	3.248	3.720	3.681	4.685
F max.	mm	16.5	13.5	15.0	17.5	16.0	15.5	17.0	15.5
	inch	.650	.531	.591	.689	.630	.610	.669	.610
G min.	mm	4.5	4.5	4.5	4.5	4.5	4.5	4.5	4.5
	inch	.177	.177	.177	.177	.177	.177	.177	.177
Preferred Limiting Dimensions									
D max.	mm	62.5	83.5	105.0	142.0	117.5	126.0	144.5	174.5
	inch	2.461	3.287	4.134	5.591	4.626	4.961	5.689	6.870
E max.	mm	41.5	59.5	77.5	83.5	84.0	96.0	95.0	120.5
	inch	1.634	2.343	3.051	3.287	3.307	3.780	3.740	4.744
F min.	mm	14.0	10.5	11.5	13.5	12.5	12.5	14.0	12.5
	inch	.551	.413	.453	.531	.492	.492	.551	.492

Liberal but unilateral loose fit tolerance permissible unless "min." and "max." is specified.
* "E" and "D" define film pack picture aperture.

* This publication is one of a series of American Standards for Photography (other than Cinematography).
† Approved June 20, 1941, by the American Standards Association, 29 West 39 Street, New York, New York. The Optical Society of America is sponsor for the project.

American Standard Dimensions Z38.1.3–1941 for 70-mm Perforated (and Unperforated) Film*†

(Cutting and Perforating Standard)



Dimensions and Tolerances

	Inches	Millimeter Equivalents
A	2.756 $^{+0.000}_{-0.004}$	70.00 $^{+0.00}_{-0.10}$
B	0.234 ±0.001	5.944±0.025
C	0.1300±0.0008	3.302±0.020
D	0.0800±0.0008	2.032±0.020
E	0.165 ±0.004	4.191±0.10
F	2.426 ±0.004	61.62 ±0.10
G	Not over 0.002	Not over 0.05
L*	23.400 ±0.030	594.36 ±0.76
R	0.016	0.406
Thickness 0.006″±0.0005″		0.1525 mm±0.0125 mm

* L = the length of any 100 consecutive perforation intervals.

Note 1: Unperforated 70-mm film shall have the same thickness and width ("A") dimensions as perforated film.
Note 2: These dimensions and tolerances apply to the material immediately after cutting and perforating.

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*This publication is one of a series of American Standards for Photography (other than Cinematography).
†Approved June 20, 1941, by the American Standards Association, 29 West 39 Street, New York, New York. The Optical Society of America is sponsor for the project.

This standard is not intended for use in the motion picture field. Its purpose is to provide a film for various scientific recording instruments such as oscillographs, electrocardiographs, seismographs and similar instruments and in its development no consideration was given to its use for motion picture purposes.

Proposed American Standard Z38.7.6 for Photographic Projection Printing Machines*†

This standard applies to enlargers for amateur and professional use, but not to special-purpose enlargers nor to cinematography.

1. Magnification

1.1 Magnification shall be defined as the ratio of a linear dimension of the projected image to the corresponding dimension of the negative.

2. Printing Equipment

2.1 Printing equipment listed for a given nominal size shall accommodate maximum standard sizes of all of the usual forms in which negative materials are furnished; i.e., roll film, cut film, film packs, etc.

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*When approved, this document will be one of a series of American Standards for Photography (other than Cinematography).

†Not Approved. Published for Trial and Criticism. The Optical Society of America is sponsor for the project.

Proposed American Recommended Practice Z38.7.7 for Photographic Projection Printing Machines*†

This recommended practice applies to enlargers for amateur and professional use, but not to special-purpose enlargers nor to cinematography.

1. Printing Equipment

1.1 The illumination on any part of the image field with any negative mask size and lens combination recommended by the manufacturer should not be less than 65 percent of the maximum illumination when measured under the following conditions:

(*a*) The enlarger should be set for the maximum magnification and all adjustments made as recommended by the manufacturer.

(*b*) If no lens is recommended by the manufacturer, a lens with a focal length approximately equal to the diagonal of the negative aperture should be used.

(*c*) If the lens has an adjustable diaphragm, its opening should be reduced until the maximum uniformity of illumination is obtained; that is, until there is no vignetting of the corner rays by the lens barrel.

(*d*) Since some convenient and commonly used photometers integrate the illumination over an appreciable area, the above recommendation is based on the assumption that the illumination for any portion of the field will be taken as the average of a circular area having a diameter of not more than one-fifth of the width of the field. For measurement of the corner illumination, this area should be tangent to the two edges of the field.

1.2 The image field should be free from local irregularities in illumination of more than 15 percent per inch (2.5 cm).

1.3 If the enlarger is equipped with an attached easel, the circle of confusion resulting from lack of squareness of the easel with respect to the lens axis should not subtend from the normal viewing distance for the reproduction, an angle of more than one minute. To maintain this recommendation, the paper support should be perpendicular to the lens axis within $0.035(m+1)f/x$ degrees.

m = minimum magnification ordinarily obtained with the equipment being tested.
f/x = aperture ratio for maximum opening of the lens.

On the same basis, the negative support should be perpendicular to the lens axis within

$$\frac{0.035(m'+1)f/x}{m'} \text{ degrees.}$$

m' = maximum magnification ordinarily obtained with the equipment being tested.

1.4 When the enlarger is operated in a manner recommended by the manufacturer, the negative should not be damaged by heat.

2. Lenses

2.1 Lenses recommended for color work should not have lateral color in excess of 0.06 percent of the semi-diagonal of the field for any wave-length of light lying between 440 and 650 mμ. For practical purposes, enlarger lenses may be checked for lateral color by the use of commercially available filters having dominant wave-lengths and transmission bands substantially as follows:

650 mμ (transmission of wave-lengths shorter than 590 mμ should be negligible).
440 mμ (370 to 510 mμ).

2.2 Enlargers sold without lenses should not require a half-angle lens covering power of more than 25 degrees to attain advertised magnifications.

3. Light Control

3.1 Printing equipment should be sufficiently light-tight so that light escaping from it will not fog bromide paper during normal handling.

3.2 Provision should be made to prevent light from passing around the edges of negatives of all sizes for which the equipment is recommended.

3.3 If a safety filter is provided, its transmission should be such that no fogging of bromide paper will occur under normal handling. The filter mount should not interfere with the image beam.

3.4 Operation of the exposure control should not cause any vibration detrimental to the print.

4. Negative Carrier

4.1 When used in the recommended manner, the negative carrier should not scratch the negative or cause other damage visible on the negative or print.

5. Interior Finish

5.1 The interior of enlargers between negative and lens and the inside of the lens mount should be finished to minimize reflection of light.

*When approved, this document will be one of a series of American Standards for Photography (other than Cinematography).

†Not Approved. Published for Trial and Criticism. December 15, 1941. The Optical Society of America is sponsor for the project.